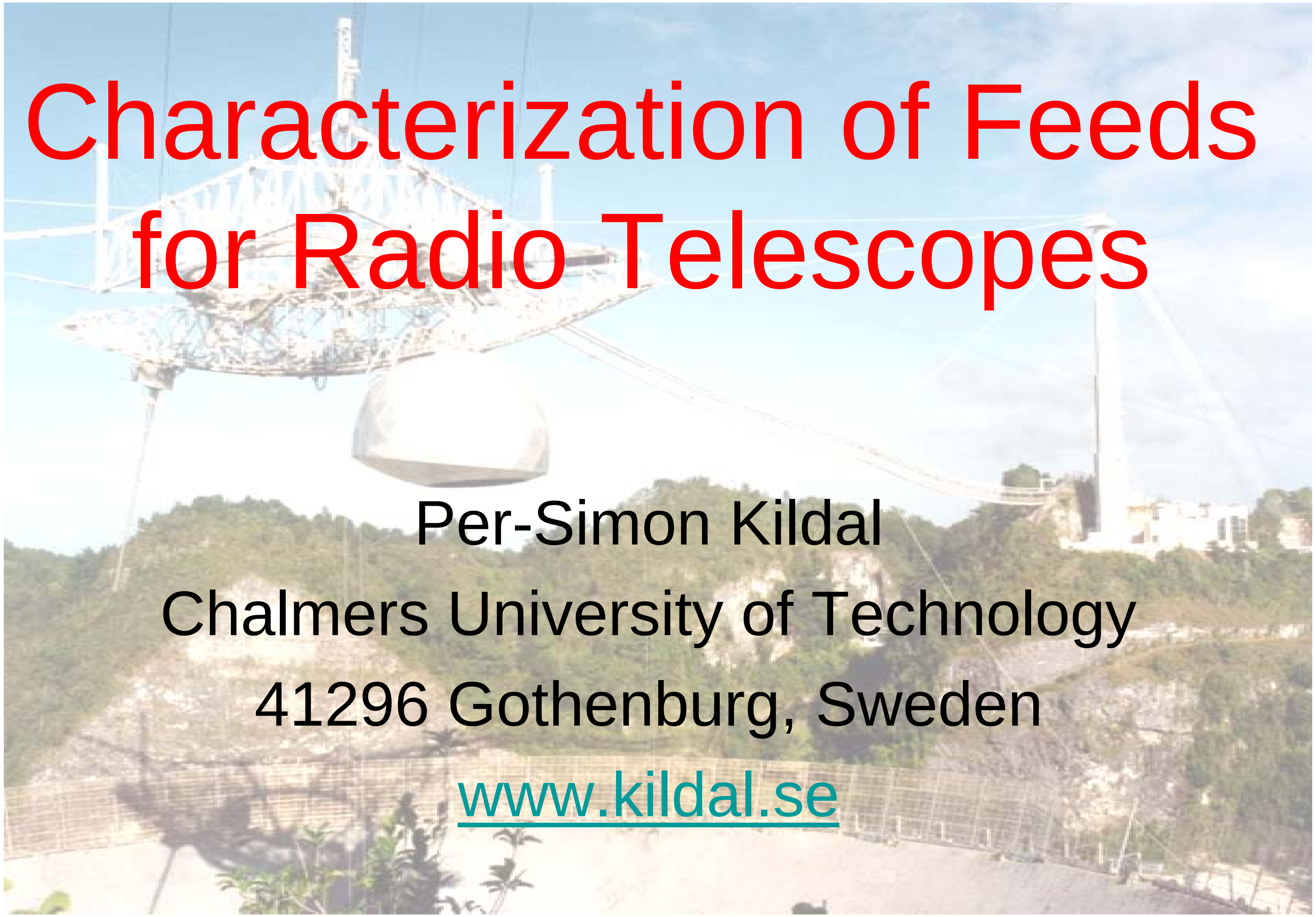
# Chalmers presentations

- Per-Simon Kildal (40 min)
  - (Tutorial) Characterization of Feeds for Radio Telescopes (including combination feeds)
- 12:30 Lunch
- Per-Simon Kildal (15min)
  - Introduction to Design of Eleven Feed for SKA and VLBI2010
  - Selection of Geometry and Port lay-out
- Leif Helldner (15min)
  - Cryogenic and Mechanical Design for the Eleven Feed
- Jian Yang (15min)
  - Numerical Optimization of log-Period Antennas and Measurements
- Per-Simon Kildal (10min)
  - Summary of Performance and What Next



# Characterization of Feeds for Radio Telescopes

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# Traditional characterization not good enough for diagnosis

- Traditionally feeds where characterized only by the feed illumination taper in E- and H-planes, and the E- and H-plane phase centers
- This is OK for traditional rotationally symmetric feeds, but NOT for modern compact or wideband feeds
- Then, we need to a better diagnosis approach

# Content of tutorial

- Radiation field function (phase ref point)
- BOR antennas and BOR1 efficiency
- Subtended half angle versus F/D
- Feed efficiency and subefficiencies
  - Polarization, Spillover, Illumination, Phase (phase center)
- Blockage and subreflector diffraction
- More efficiencies
  - Mismatch factor, radiation efficiency
- How do they affect G/Tsys
- Combination feeds and decoupling efficiency

# Radiation field and radiation field function

General form:

$$\boldsymbol{E}(r, \theta, \varphi) = \frac{1}{r} e^{-jkr} \boldsymbol{G}(\theta, \varphi)$$

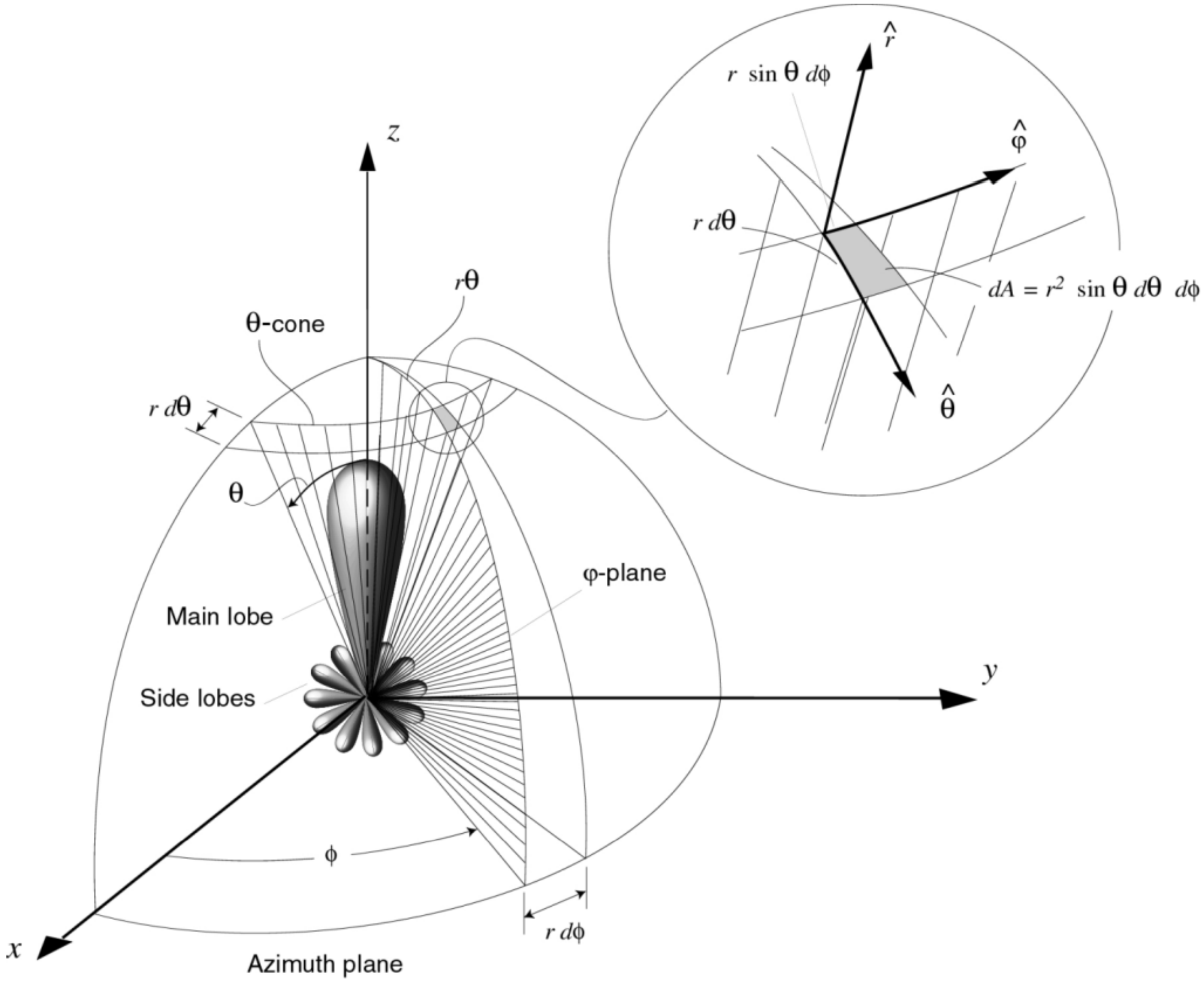
Divergence factor

Phase factor

Complex radiation Field function

When $r = \sqrt{x^2 + y^2 + z^2}$, the phase reference point of $\mathbf{G}(\theta, \varphi)$ is origin (0,0,0)

# Spherical coordinate system




$$\mathbf{r} = r\sin\theta\cos\varphi\,\hat{\mathbf{x}} + r\sin\theta\sin\varphi\,\hat{\mathbf{y}} + r\cos\theta\,\hat{\mathbf{z}}$$

# Equivalent forms of radiation field and its function

$$\mathbf{E}(\mathbf{r}) = \frac{1}{r} e^{-jkr} \boldsymbol{G}(\hat{\mathbf{r}})$$

$$\boldsymbol{E}(r, \theta, \varphi) = \frac{1}{r} e^{-jkr} \boldsymbol{G}'(\theta, \varphi)$$

$$\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}} = \hat{\mathbf{r}} r$$

$$\mathbf{r} = r\sin\theta\cos\varphi\,\hat{\mathbf{x}} + r\sin\theta\sin\varphi\,\hat{\mathbf{y}} + r\cos\theta\,\hat{\mathbf{z}}$$

$$\boldsymbol{G}'(\theta, \varphi) = \boldsymbol{G}(\hat{\mathbf{r}})$$

direction $\hat{\mathbf{r}}$

# Polarization and polarization vectors

Co- and cross-polar radiation field functions:

$$G_{co}(\theta, \varphi) = \boldsymbol{G}(\theta, \varphi) \cdot \hat{\boldsymbol{co}}^*(\theta, \varphi)$$

$$G_{xp}(\theta, \varphi) = \boldsymbol{G}(\theta, \varphi) \cdot \hat{\boldsymbol{xp}}^*(\theta, \varphi)$$

Convenient choice for
**linear y-polarization, i.e.**
**Ludwig's third definition:**

$$\hat{\boldsymbol{co}}(\theta, \varphi) = \hat{\boldsymbol{y}}'(\theta, \varphi) = \sin\varphi\hat{\theta} + \cos\varphi\hat{\varphi}$$

$$\hat{\boldsymbol{xp}}(\theta, \varphi) = \hat{\boldsymbol{x}}'(\theta, \varphi) = \cos\varphi\hat{\theta} - \sin\varphi\hat{\varphi}$$

**For RHC pol.:**

$$\hat{\boldsymbol{co}}(\theta, \varphi) = [\hat{\boldsymbol{x}}'(\theta, \varphi) - j\hat{\boldsymbol{y}}'(\theta, \varphi)]/\sqrt{2} = e^{-j\phi}[\hat{\theta} - j\hat{\varphi}]/\sqrt{2}$$

$$\hat{\boldsymbol{xp}}(\theta, \varphi) = [\hat{\boldsymbol{x}}'(\theta, \varphi) + j\hat{\boldsymbol{y}}'(\theta, \varphi)]/\sqrt{2} = e^{j\phi}[\hat{\theta} + j\hat{\varphi}]/\sqrt{2}$$

# Fourier expansion of radiation field function

- The azimuth variation of the pattern can be expanded in a Fourier series in two ways, :

$$\boldsymbol{G}(\theta, \varphi) = G_{co}(\theta, \varphi)\hat{\boldsymbol{co}} + G_{xp}(\theta, \varphi)\hat{\boldsymbol{xp}} =$$

$$= \sum_{n=1}^{\infty} [CO_{sn}(\theta)\sin(n\varphi) + CO_{cn}(\theta)\cos(n\varphi)]\hat{\boldsymbol{co}}$$

$$+ \sum_{n=1}^{\infty} [XP_{sn}(\theta)\sin(n\varphi) + XP_{cn}(\theta)\cos(n\varphi)]\hat{\boldsymbol{xp}}$$

- Alternative which is consistent with spherical TE and TM mode expansions (note that $\boxed{A_n(0) = B_n(0) = C_n(0) = D_n(0) = 0 \quad for \quad \theta = 0 \; except \; when \; n = 1}$ )

$$\boxed{\begin{aligned} &\mathbf{G}(\theta, \varphi) = G_\theta(\theta, \varphi)\hat{\theta} + G_\varphi(\theta, \varphi)\hat{\varphi} = \\ &\sum_{n=1}^{\infty} [A_n(\theta)\sin(n\varphi) + B_n(\theta)\cos(n\varphi)]\hat{\theta} + \sum_{n=1}^{\infty} [C_n(\theta)\cos(n\varphi) - D_n(\theta)\sin(n\varphi)]\hat{\varphi} \end{aligned}}$$

# Power integral

- Total radiated power:

$$P_{rad} = \frac{1}{2\eta} \int\int_{4\pi} [|G_\varphi(\theta,\varphi)| + |G_\theta(\theta,\varphi)| ] \sin\theta \, d\theta \, d\varphi$$

- We will define a power integral:

$$P = \int\int_{4\pi} [|G_\varphi(\theta,\varphi)|^2 + |G_\theta(\theta,\varphi)|^2] \sin\theta \, d\theta \, d\varphi$$

- The power integral can also be calculated by:
  - aperture integration (for some aperture antennas)
  - integration over feed pattern (in reflector antennas)

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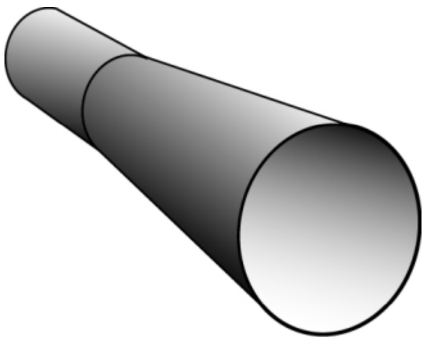
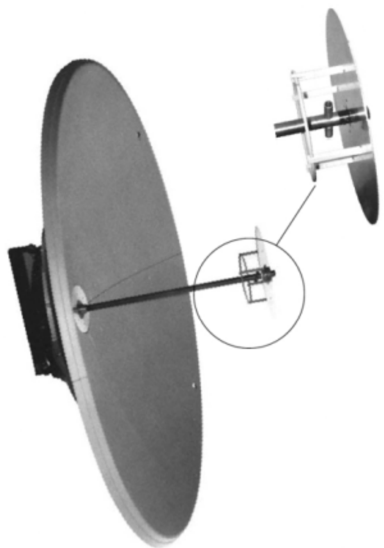
# Directivity and BOR1 efficiency

- Power integral: $$P = \sum_{all\ n} P_n$$
- Directivity $$D = \frac{4\pi \left| G_{co}(0) \right|^2}{P} = e_{BOR1} D_{BOR1}$$
- BOR1 efficiency $$e_{BOR1} = \frac{P_1}{P}$$
- BOR1 directivity $$D_{BOR1} = \frac{4\pi \left| G_{co}(0) \right|^2}{P_1}$$

# BOR antennas

- We use the term
  BOR (Bodies Of Revolution) antennas
  to characterize antennas which material parts are completely rotationally symmetric around the same (z-)axis.
- The field can still have an azimuth variation. The order of this variation is characterized by an index on BOR.
- The abbreviation BOR is well known in MoM analysis.





Reflector antenna with dipole-disk feed

Spiral wire antenna on corrugated disk

Corrugated horn antenna

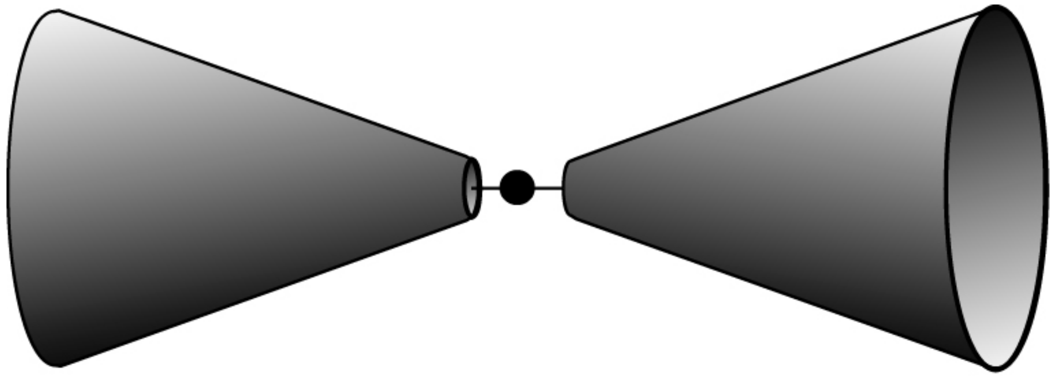
# $BOR_0$ antennas

- BOR = Body of revolution
- $BOR_0$ antennas (no azimuth variation):
- Electric dipole excited:
- Magnetic dipole excited:

$$\boldsymbol{G}_e(\theta, \varphi) = B_0(\theta)\hat{\theta}$$

$$\boldsymbol{G}_m(\theta, \varphi) = C_o(\theta)\hat{\varphi}$$

- Example:



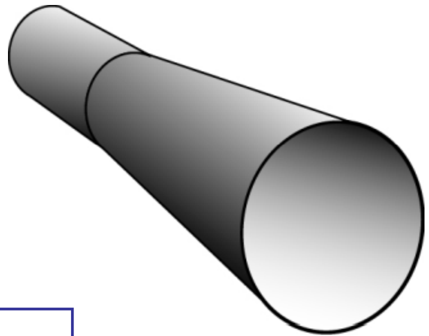
Biconical $BOR_0$ antenna

# $\mathrm{BOR}_1$ antennas

## (excited for single phi-variation only)

- BOR with y-polarized $\mathrm{TE}_{11}$ type of excitation:

$$\boldsymbol{G}(\theta,\varphi) = \underbrace{G_E(\theta)}_{\text{E-plane pattern}}\sin\varphi\,\hat{\theta} + \underbrace{G_H(\theta)}_{\text{H-plane pattern}}\cos\varphi\,\hat{\varphi}$$



- Co- and crosspolar components:

$$G_{co}(\theta,\varphi) = \boldsymbol{G}_y(\theta,\varphi)\cdot\hat{\boldsymbol{co}}^* = G_{co45}(\theta) - G_{xp45}(\theta)\cos 2\varphi$$
$$G_{xp}(\theta,\varphi) = \boldsymbol{G}_y(\theta,\varphi)\cdot\hat{\boldsymbol{xp}}^* = G_{xp45}(\theta)\sin 2\varphi$$

- Co- and crosspolar components in 45 deg plane:

$$G_{co45}(\theta) = \frac{1}{2}[G_E(\theta) + G_H(\theta)]$$
$$G_{xp45}(\theta) = \frac{1}{2}[G_E(\theta) - G_H(\theta)]$$

The BOR1 relations

We can construct the whole radiation pattern if we know the co- and crosspolar patterns in the 45 deg plane, or the E- and H-plane patterns.

# BOR$_1$ antenna: Circular pol. (RHC)

- RHC co- and cross-polar patterns become:

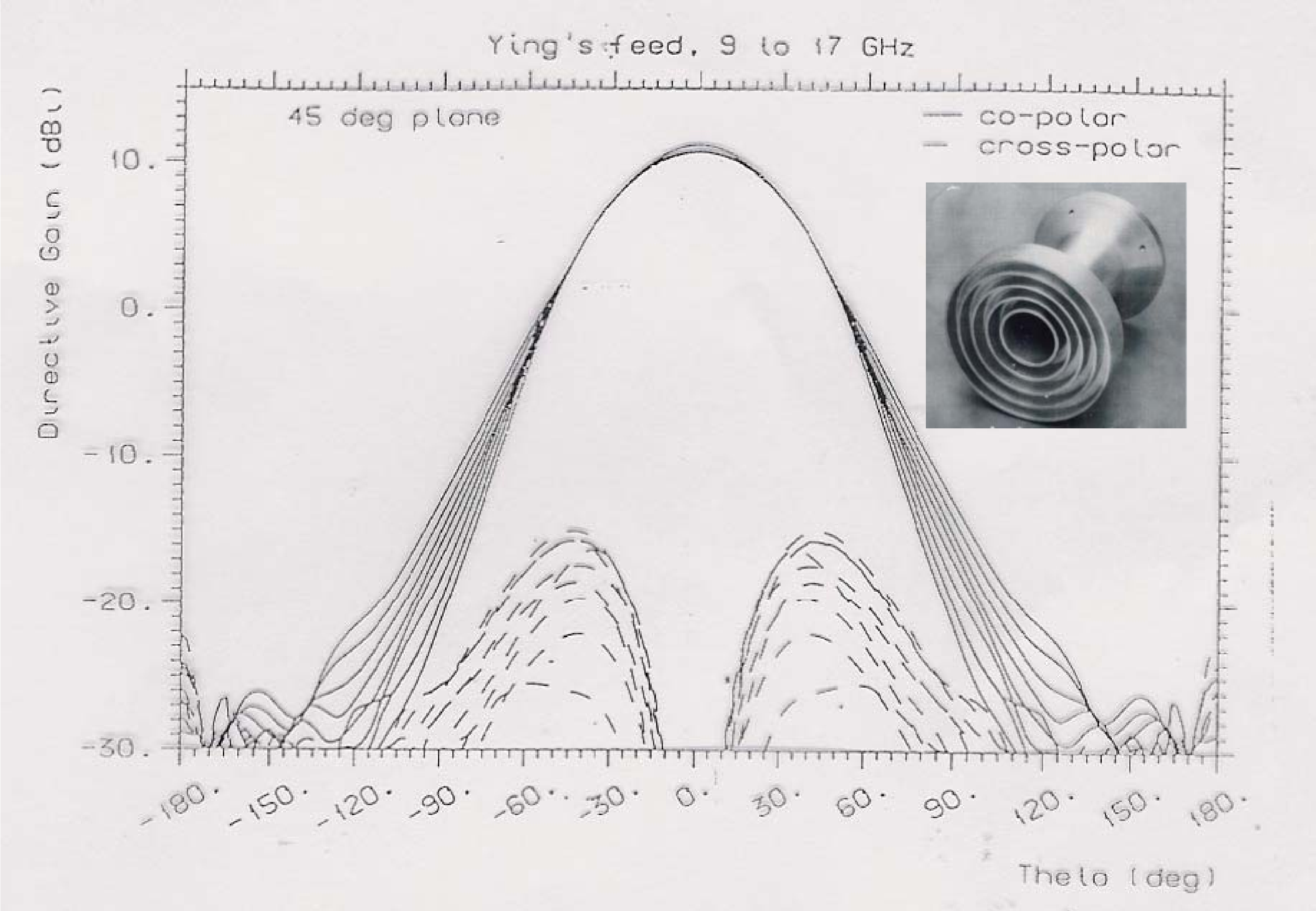
$$\boxed{\begin{aligned} G_{co}(\theta,\varphi) &= \boldsymbol{G}_c \cdot \hat{\boldsymbol{co}}^* = G_{co45}(\theta) \\ G_{xp}(\theta,\varphi) &= \boldsymbol{G}_c \cdot \hat{\boldsymbol{xp}}^* = G_{xp45}(\theta)e^{-j2\varphi} \end{aligned}}$$

- Where as before:

$$\boxed{G_{co45}(\theta) = \frac{1}{2}[G_E(\theta) + G_H(\theta)]} \qquad \boxed{G_{xp45}(\theta) = \frac{1}{2}[G_E(\theta) - G_H(\theta)]}$$

The co- and xp- 45 deg patterns for linear polarization are equal to the co- and xp-patterns in all planes for circular polarization. Useful relation, but valid only for ideal BOR1 antennas.

# SOFT choke horn feed: Constant beamwidth over 0.9-1.7 GHz (Ying, Kishk, Kildal, 1995)



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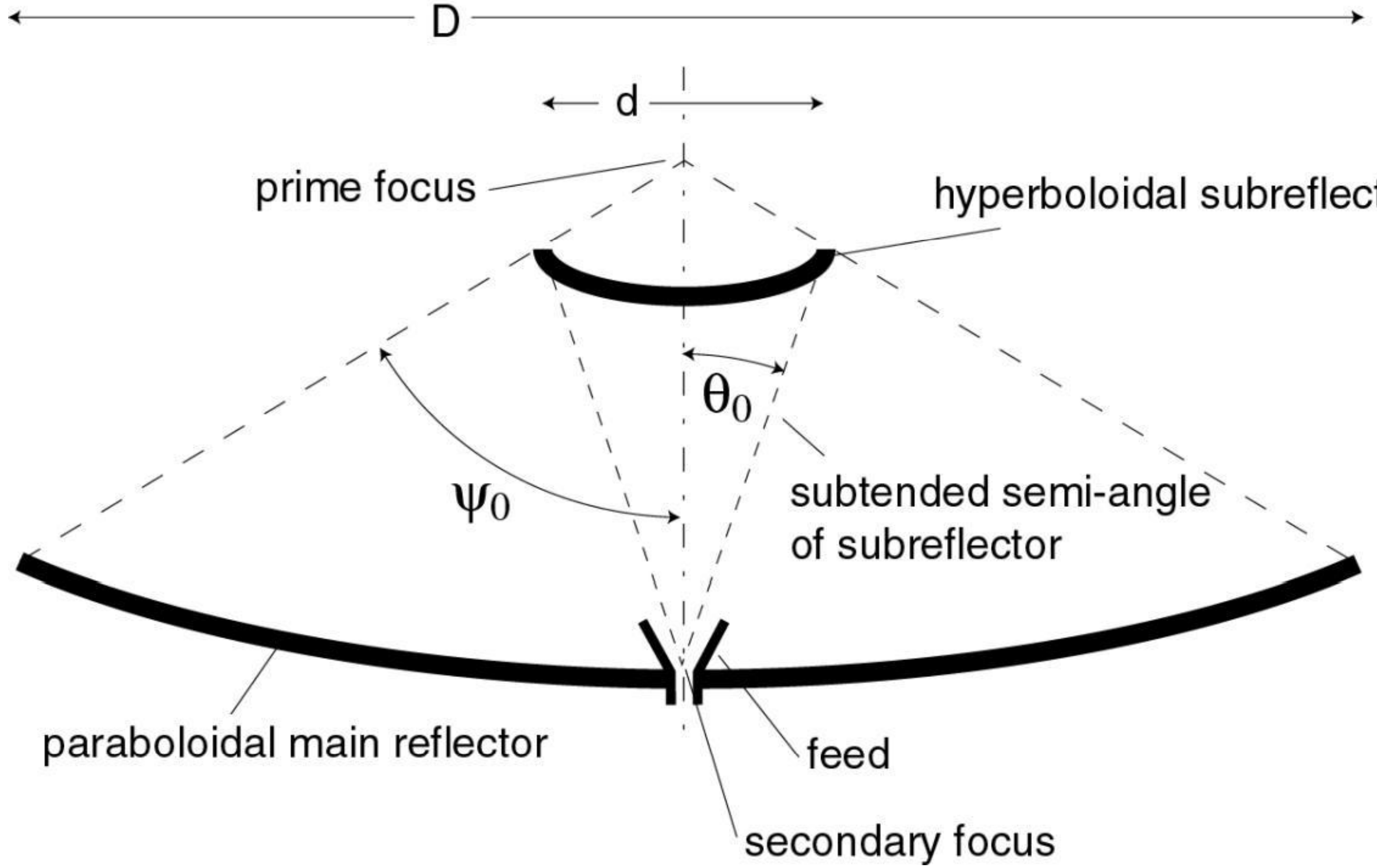
# Cassegrain antenna

Paraboloidal main reflector and hyperbloidal subreflector.
Four independent parameters $D, d, \theta_0, \psi_0$
Primary focal point, secondary focal point.



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# Paraboloid: Aperture or feed efficiency

$$e_{ap} = \frac{4\pi\cot^2(\theta_0/2)\left|\int_0^{\theta_0} G_{co45}(\theta_f)\tan(\theta_f/2)\,d\theta_f\right|^2}{2\pi\int_0^{\pi}\left[\left|G_{co45}(\theta_f)\right|^2 + \left|G_{xp45}(\theta_f)\right|^2\right]\sin\theta_f\,d\theta_f}$$

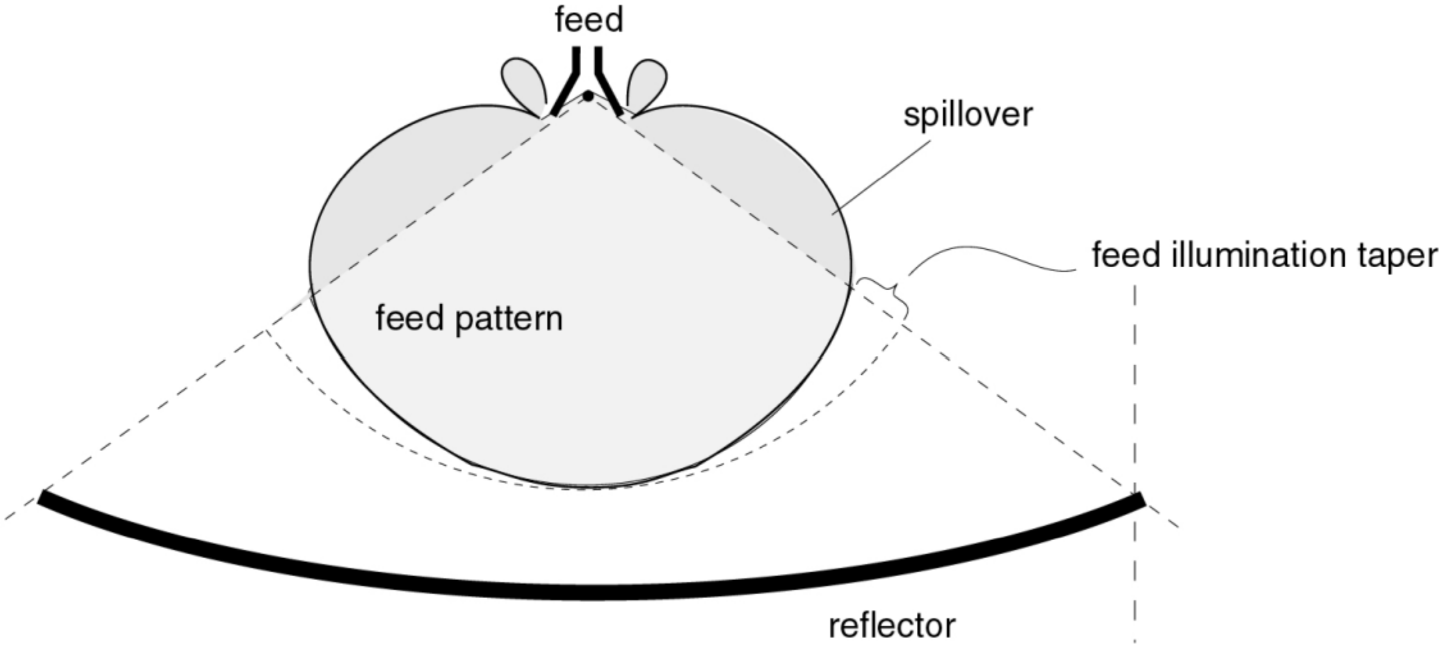
Alternative expression based on aperture integration:

$$e_{ap} = e_{sp}\left(\frac{\left(\frac{1}{A}\right)\left|2\pi\int_0^{D/2} E_{co45}(\rho)\rho\,d\rho\right|^2}{2\pi\int_0^{D/2}\left[\left|E_{co45}(\rho)\right|^2 + \left|E_{xp45}(\rho)\right|^2\right]\rho\,d\rho}\right)$$

Similar expressions exist for reflector antennas of other forms.

# Paraboloid: Spillover efficiency

$$e_{sp} = \frac{2\pi \int_0^{\theta_0} \left[ \left| G_{co45}(\theta_f) \right|^2 + \left| G_{xp45}(\theta_f) \right|^2 \right] \sin\theta_f \, d\theta_f}{2\pi \int_0^{\pi} \left[ \left| G_{co45}(\theta_f) \right|^2 + \left| G_{xp45}(\theta_f) \right|^2 \right] \sin\theta_f \, d\theta_f}$$



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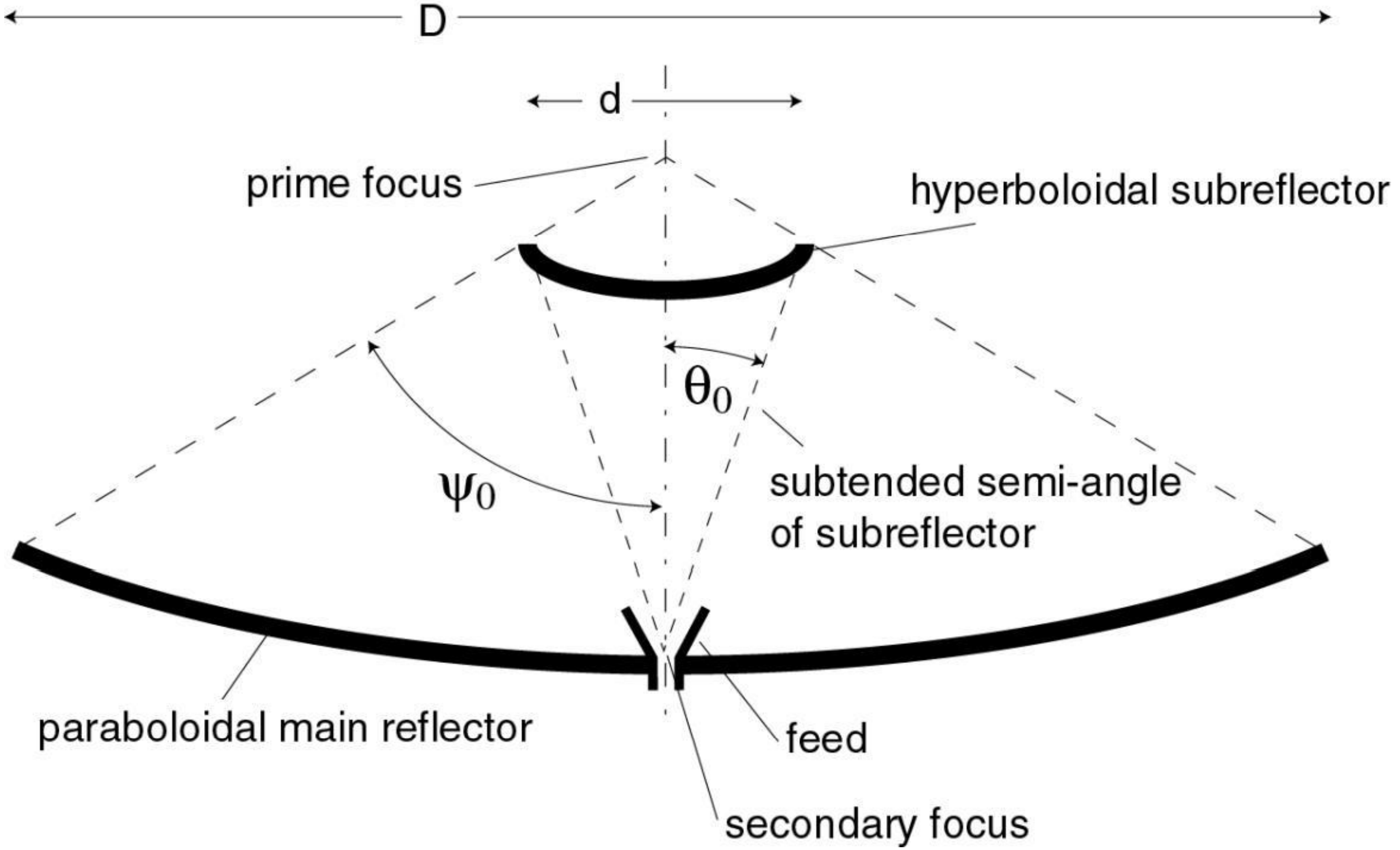


# Subefficiencies of Paraboloids and Cassegrain Antennas

Similar formulas apply to general multi-reflector systems.

Factorization of feed efficiency: $e_{ap} = e_{sp} e_{pol} e_{ill} e_{\phi}$

Spillover, polarization, illumination and phase eff.

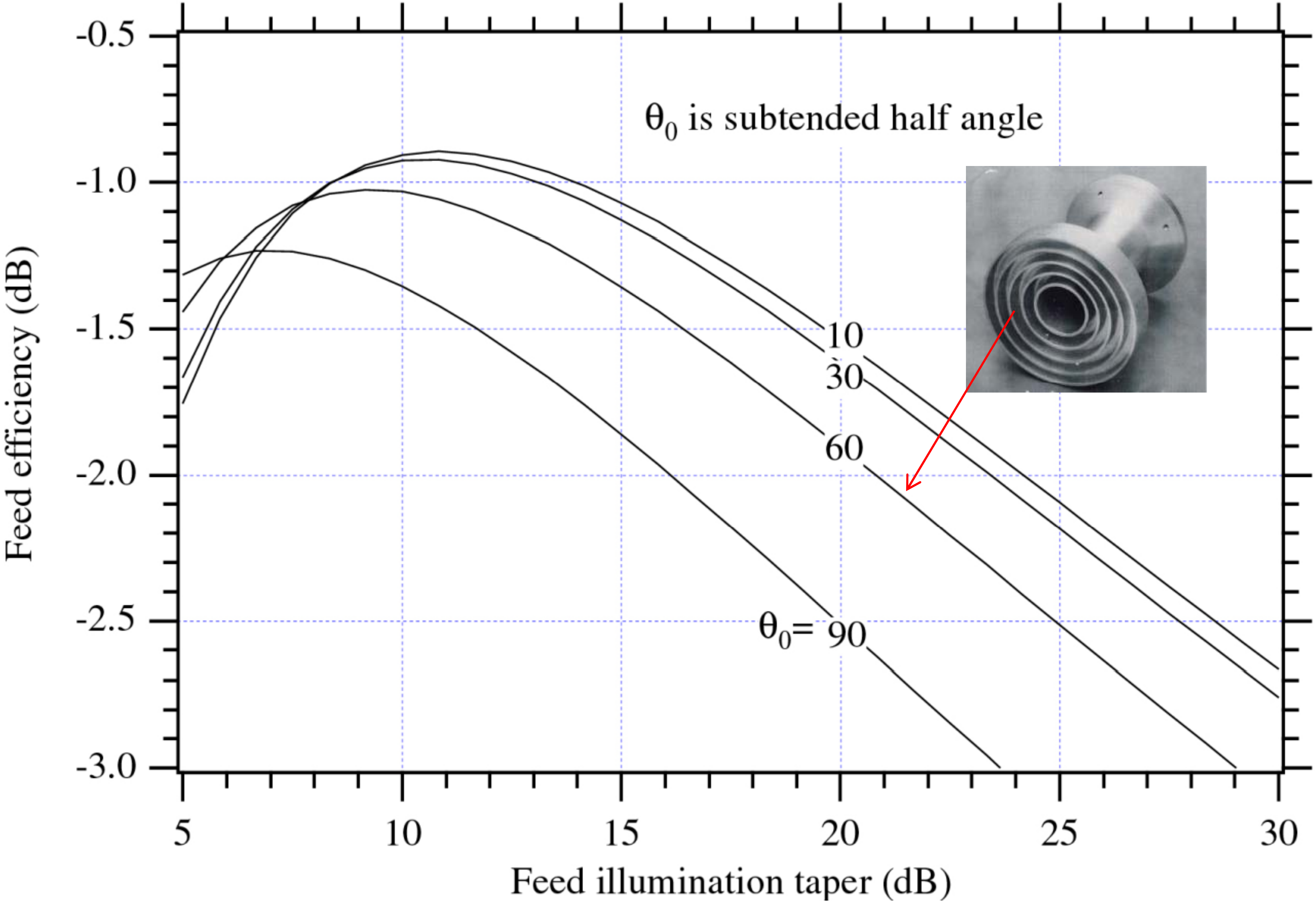
Spillover efficiency $e_{sp}$

Relative spillover power is given by $1 - e_{sp}$

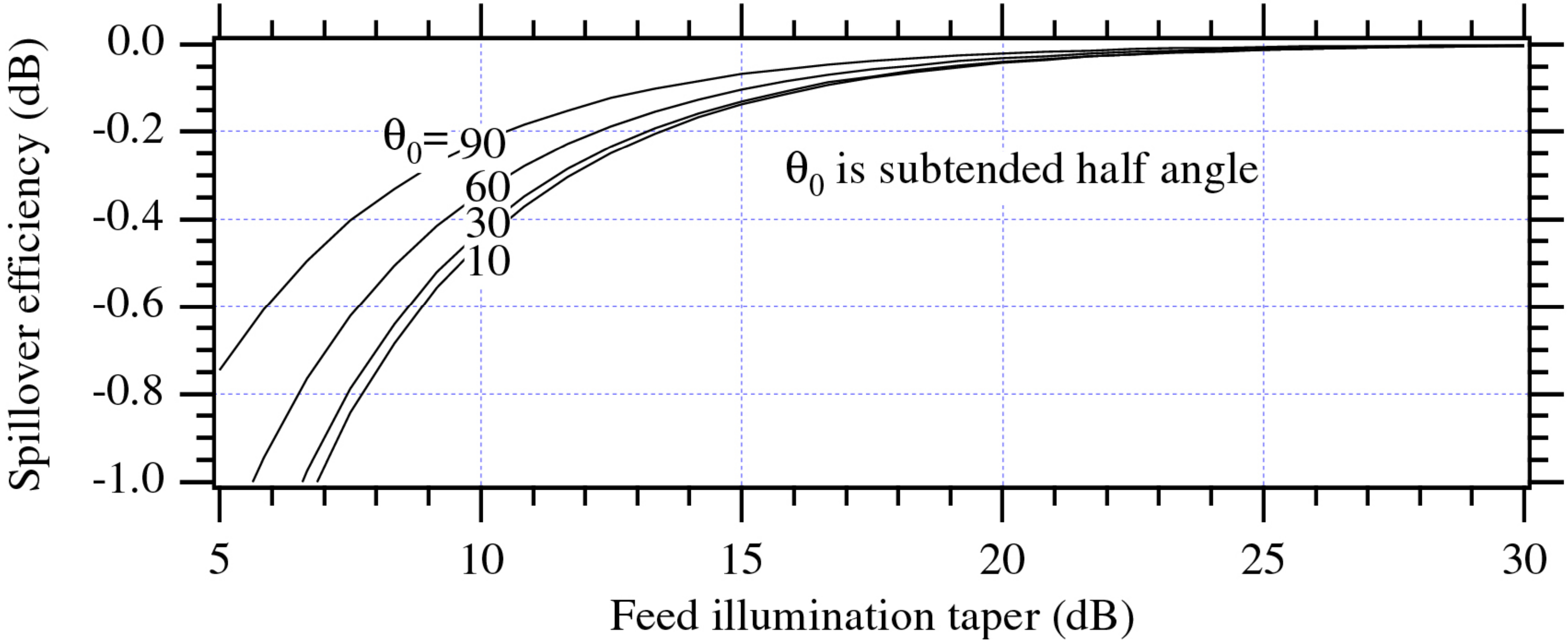
Typically between -0.05 dB and -0.5 dB.

Major contributor to the antenna noise temperature.

# Paraboloid and Cassegrain: Feed efficiency versus taper



# Paraboloid and Cassegrain: Spillover versus taper



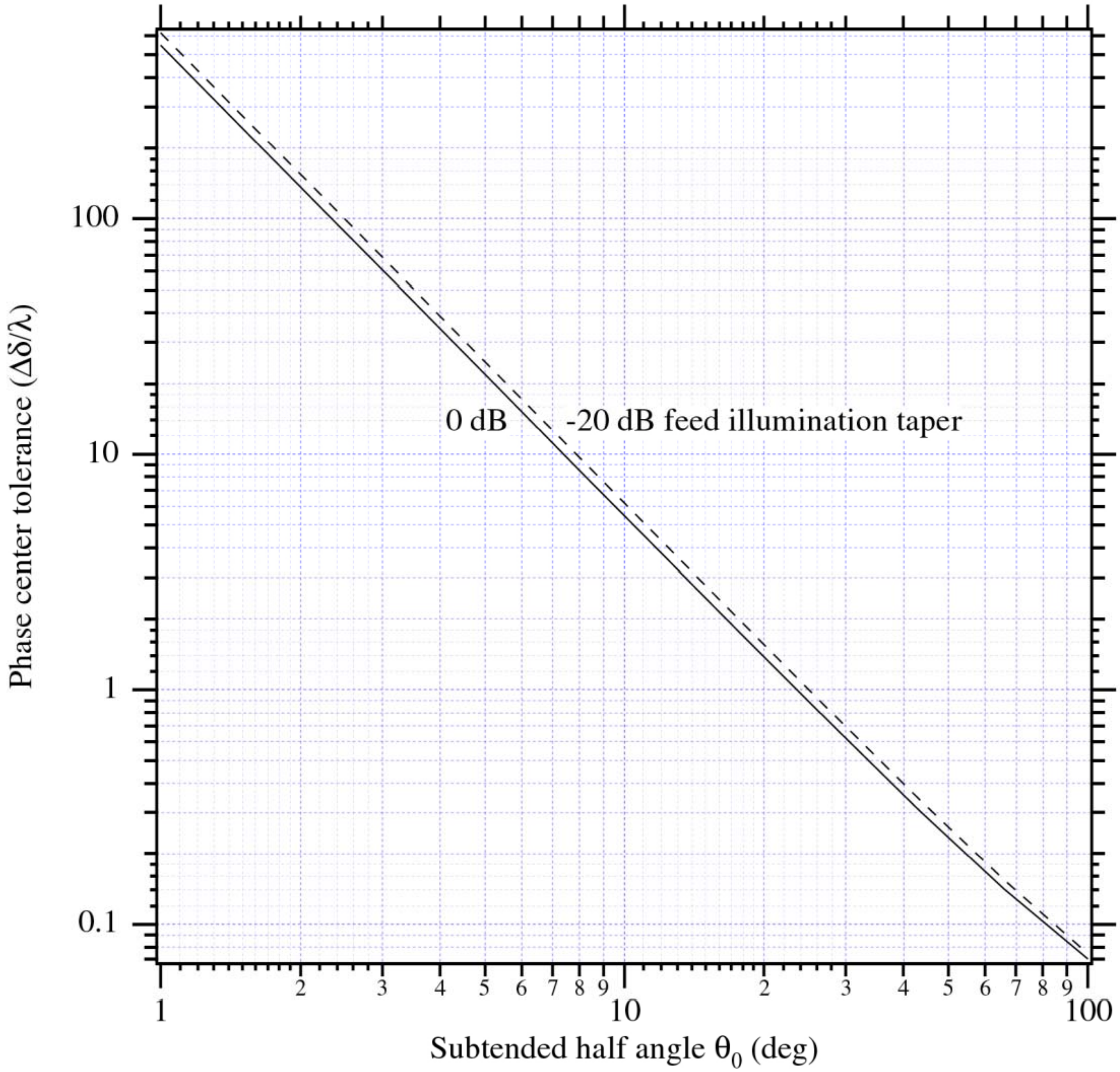
# Paraboloid and Cassegrain: Phase efficiency

$$e_\phi = \frac{\left| \int_0^{D/2} E_{co45}(\rho)\,\rho\, d\rho \right|^2}{\left[ \int_0^{D/2} \left| E_{co45}(\rho) \right| \rho\, d\rho \right]^2} = \frac{\left| \int_0^{\theta_0} G_{co45}(\theta_f) \tan(\theta_f/2)\, d\theta_f \right|^2}{\left[ \int_0^{\theta_0} \left| G_{co45}(\theta_f) \right| \tan(\theta_f/2)\, d\theta_f \right]^2}$$

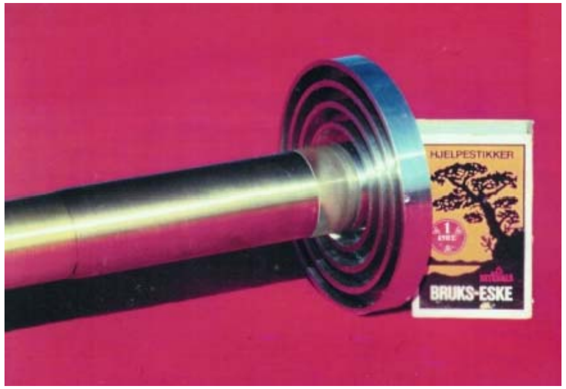
Typically better than 0.1 dB.

Use to define a <u>phase center</u> for the feed
As the phase reference point that maximizes phas efficiency.

Paraboloid and Cassegrain: Maximum displacement of phase center from focal point (for -0.1 dB phase eff.) versus subtended half angle

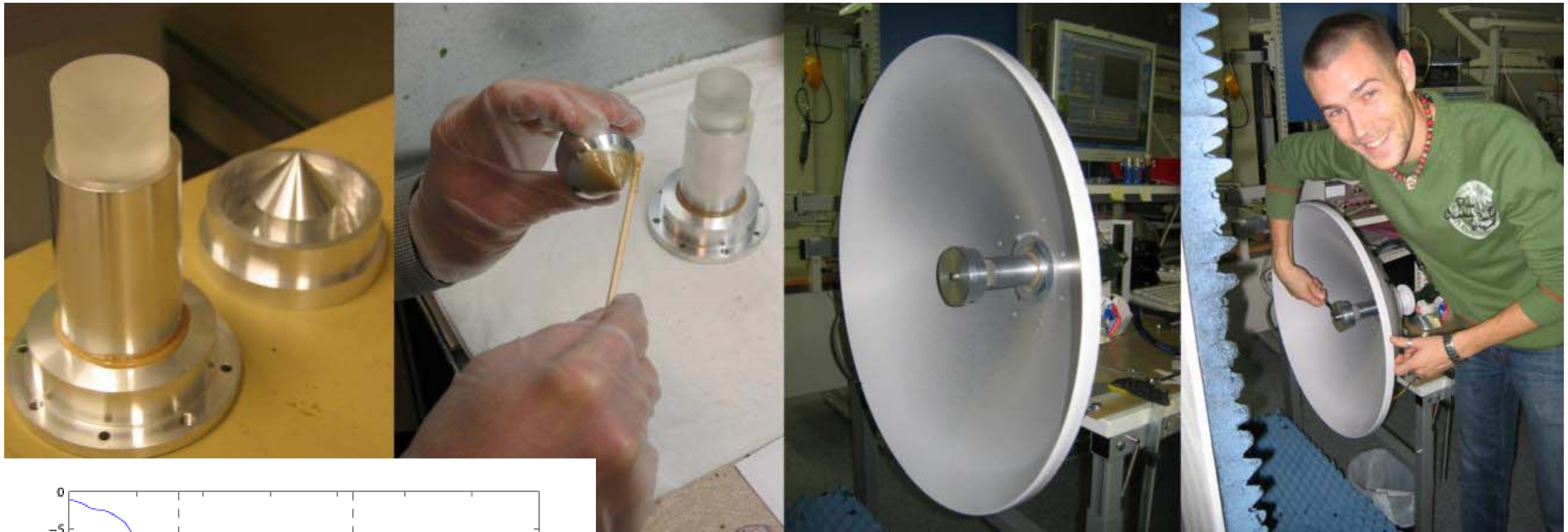


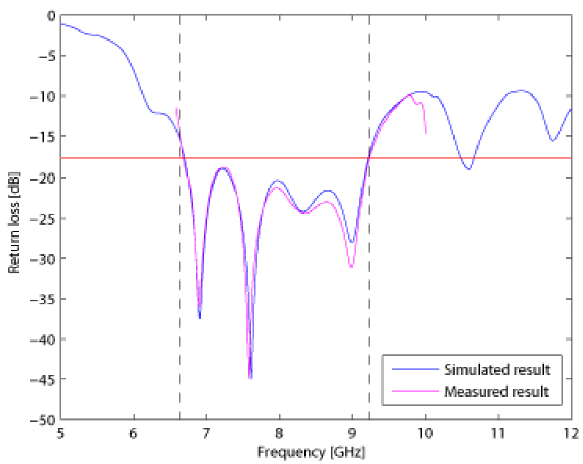
# Hat feed in ring-focus paraboloid 1986




Low sidelobes
Good efficiency

# 2007: Master student Denstedt improved bandwidth of hat feed from 10 to 32%

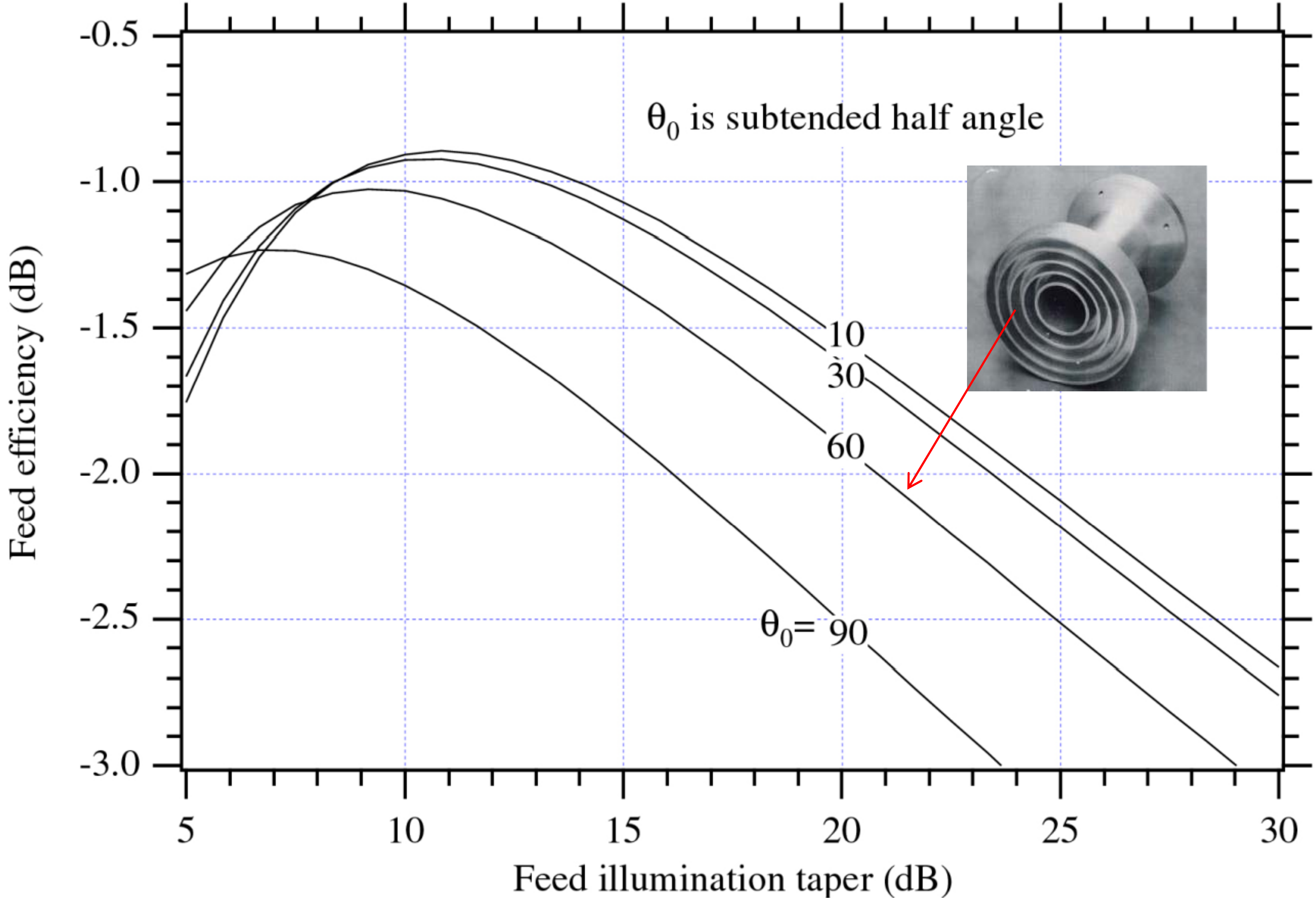




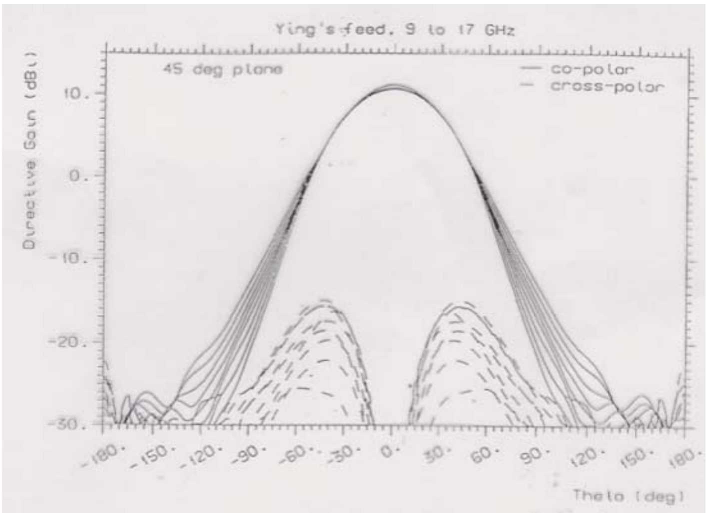

Inherently this type of antenna has very low polarization and phase efficiency.
Reflector shaped for high phase efficiency.
Hat tuned for both good impedance match and high polarization efficiency.

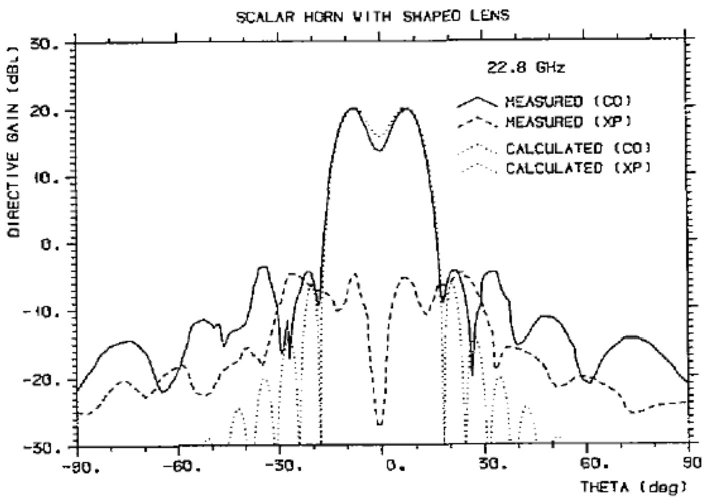
COMHAT

# Paraboloid and Cassegrain: Feed efficiency versus taper

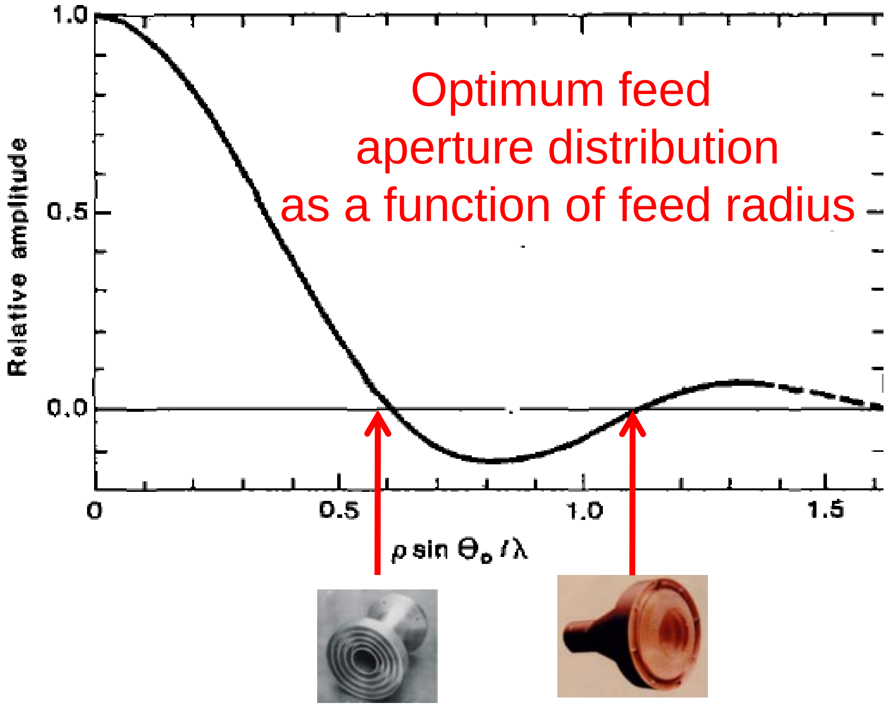


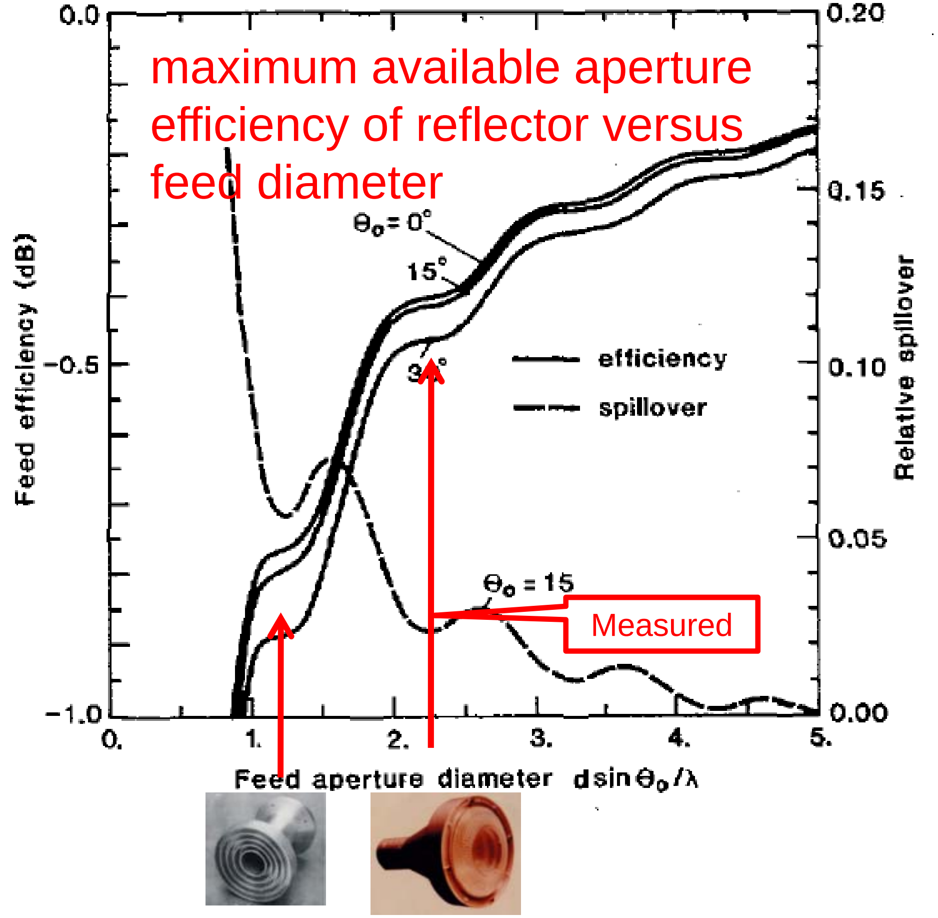
# Improved sector-shaped radiation pattern of horn with shaped lens



The horn with shpaed lens should give 0.5 dB higher illumunation efficiency than a horn with a normal feed pattern, but it did not.



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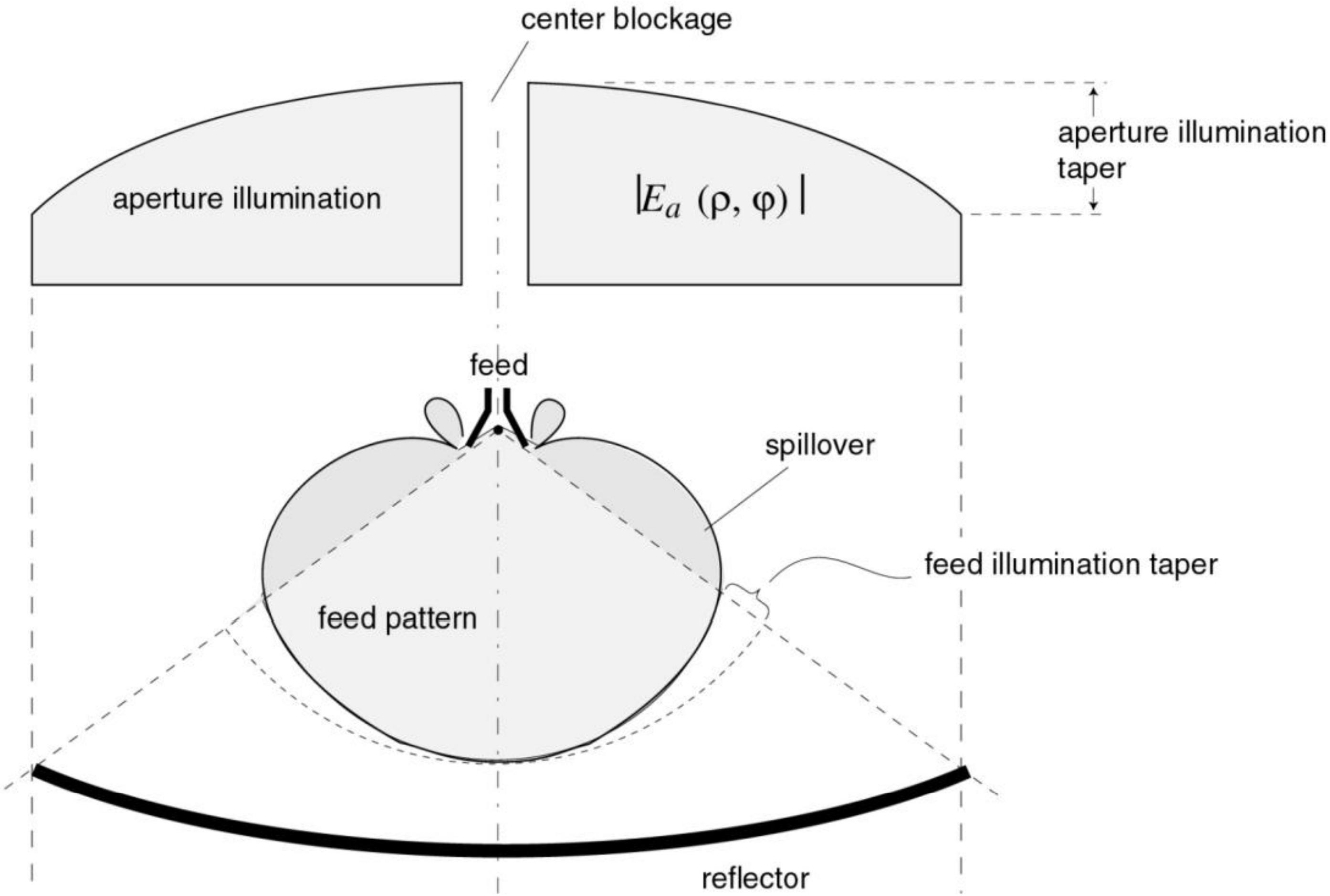
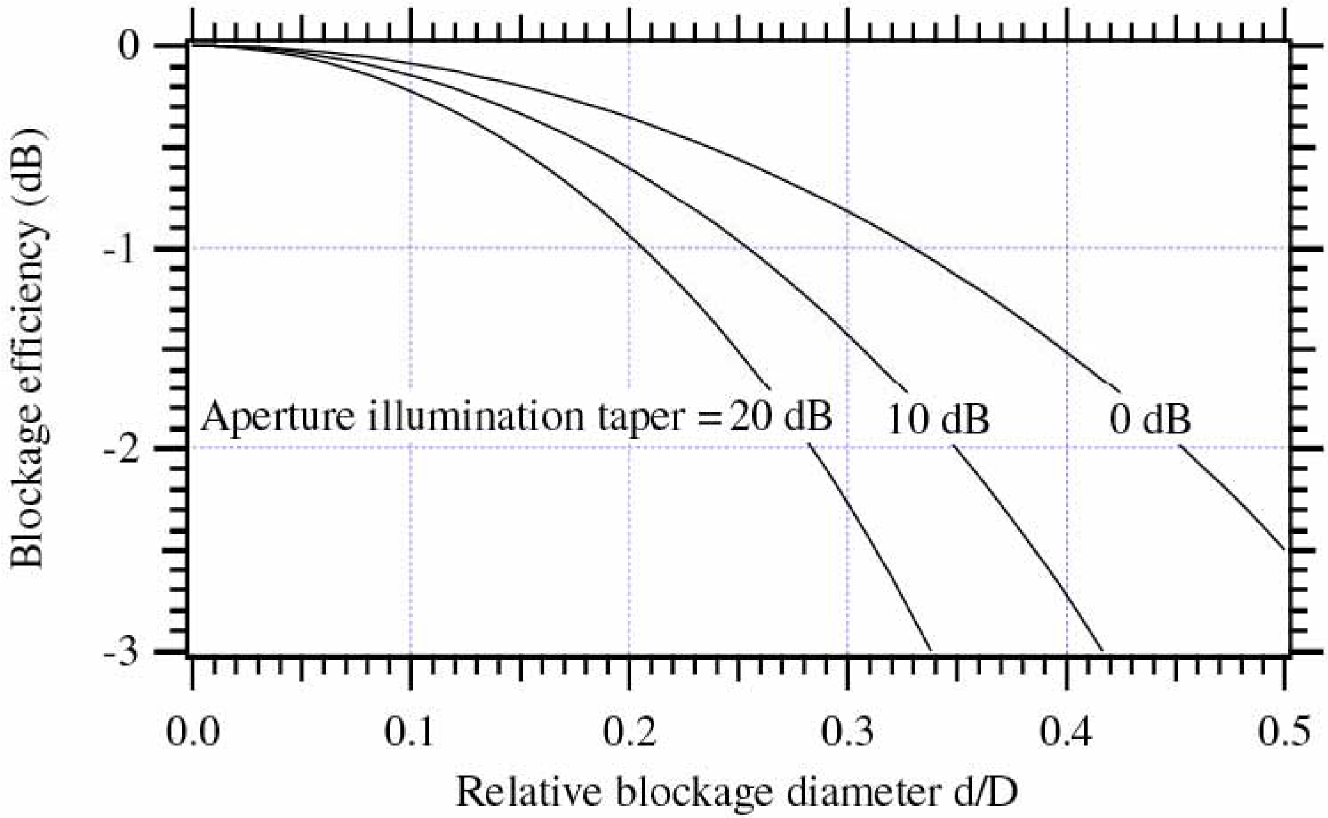
# Geometrical aperture blockage model is often used, but does not work in small reflectors, see next slide



# Blockage efficiency

The total efficiency with blockage can be written:

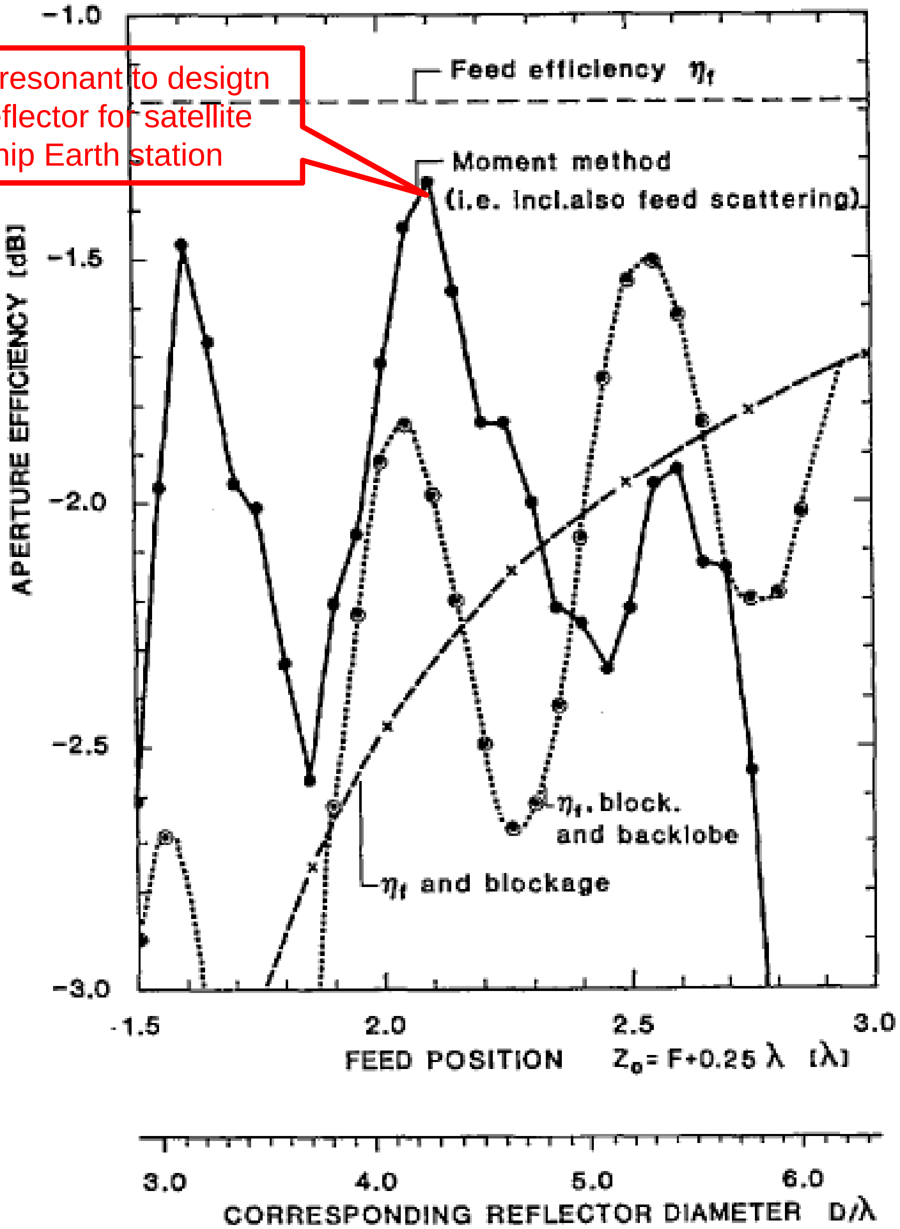
$$(e_{ap})_b = e_{ap}\left|1-\Delta_{cb}\right|^2 \qquad \Delta_{cb} = C_b(d/D)^2$$



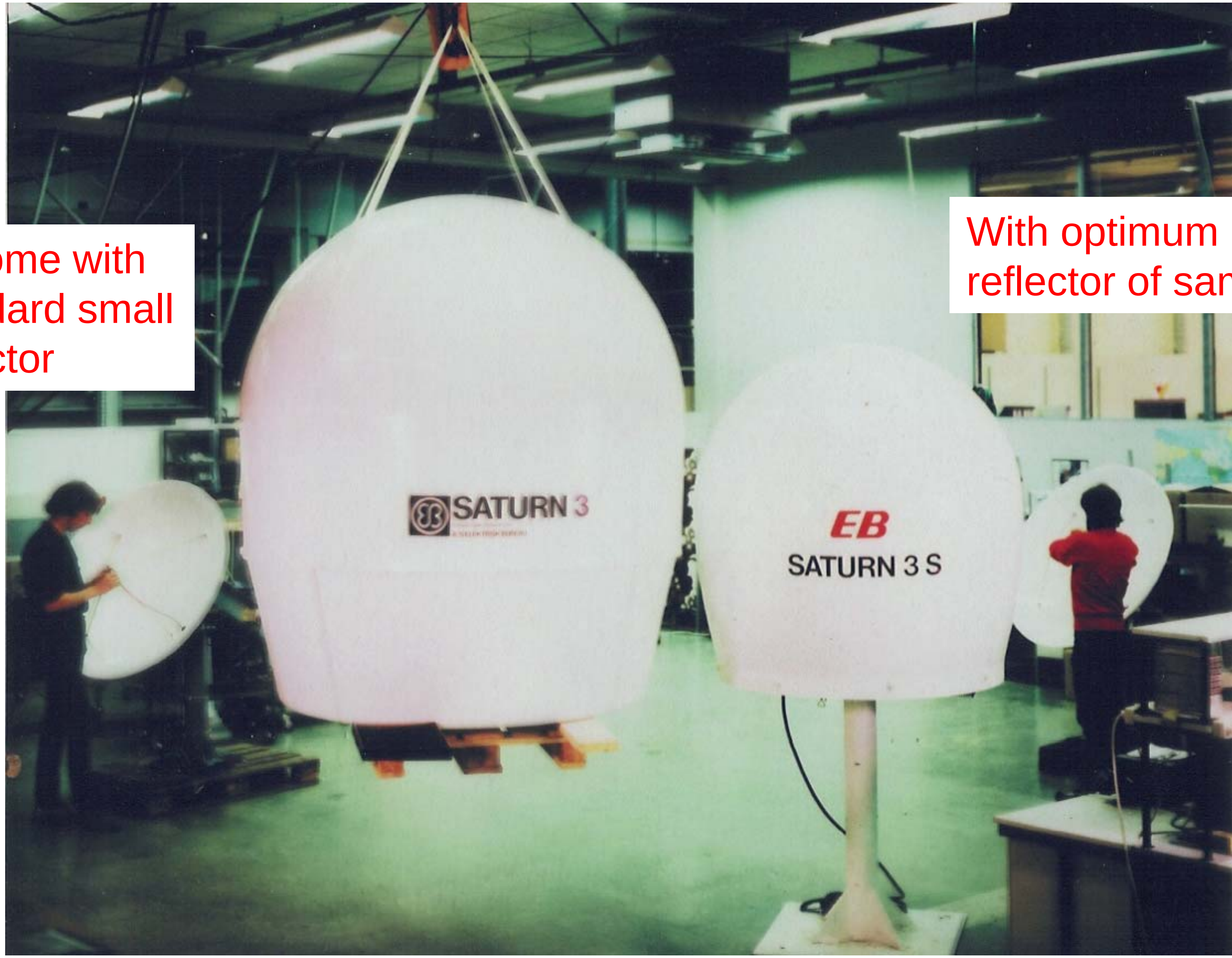

Multiple reflections and standing waves will moderate these values

In small primary-fed reflectors multiple reflections between feed and reflector can be used to increase gain

This resonant to desigtn a reflector for satellite ship Earth station




# Resonant reflectors can be very efficient and influence system design strongly




Radome with standard small reflector

With optimum resonant reflector of same gain

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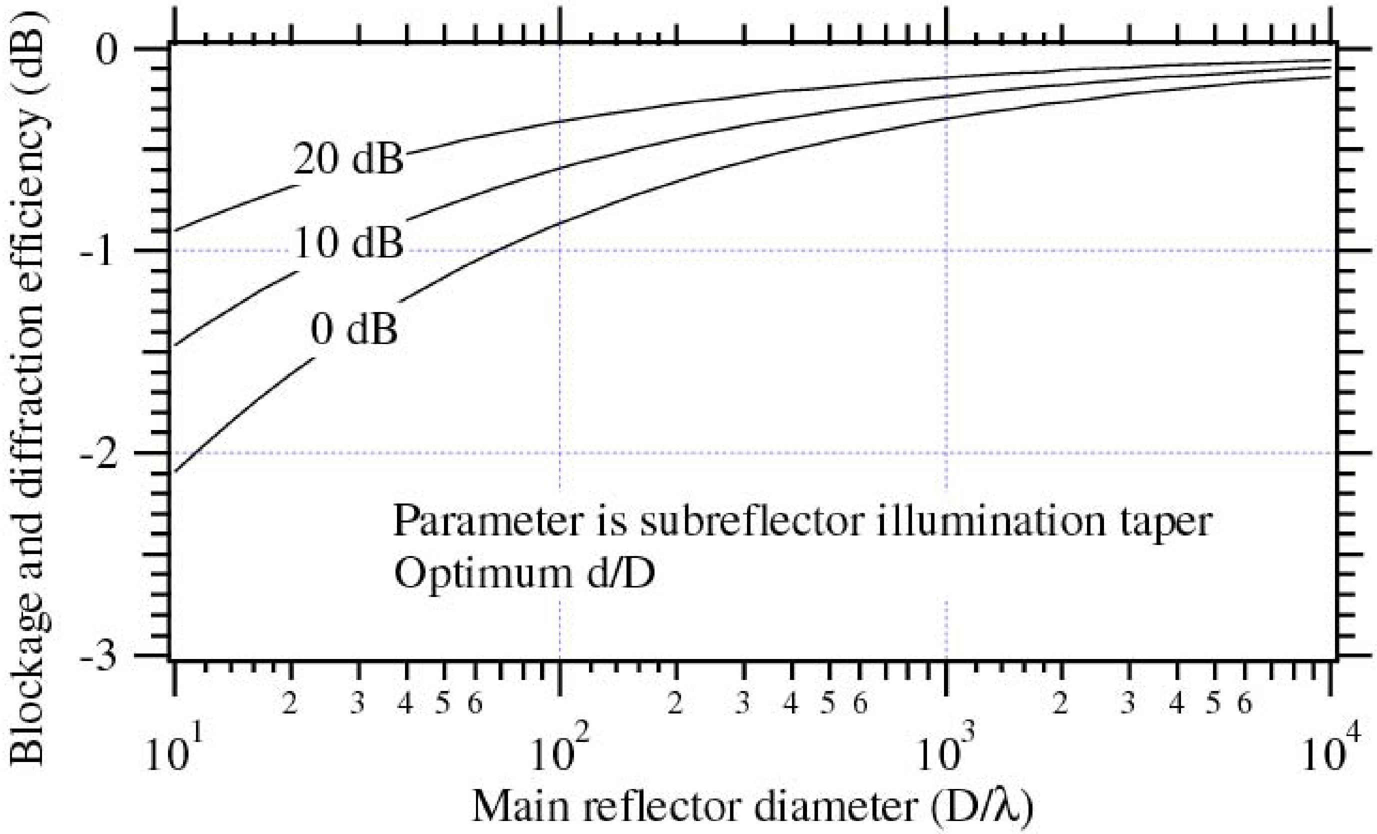
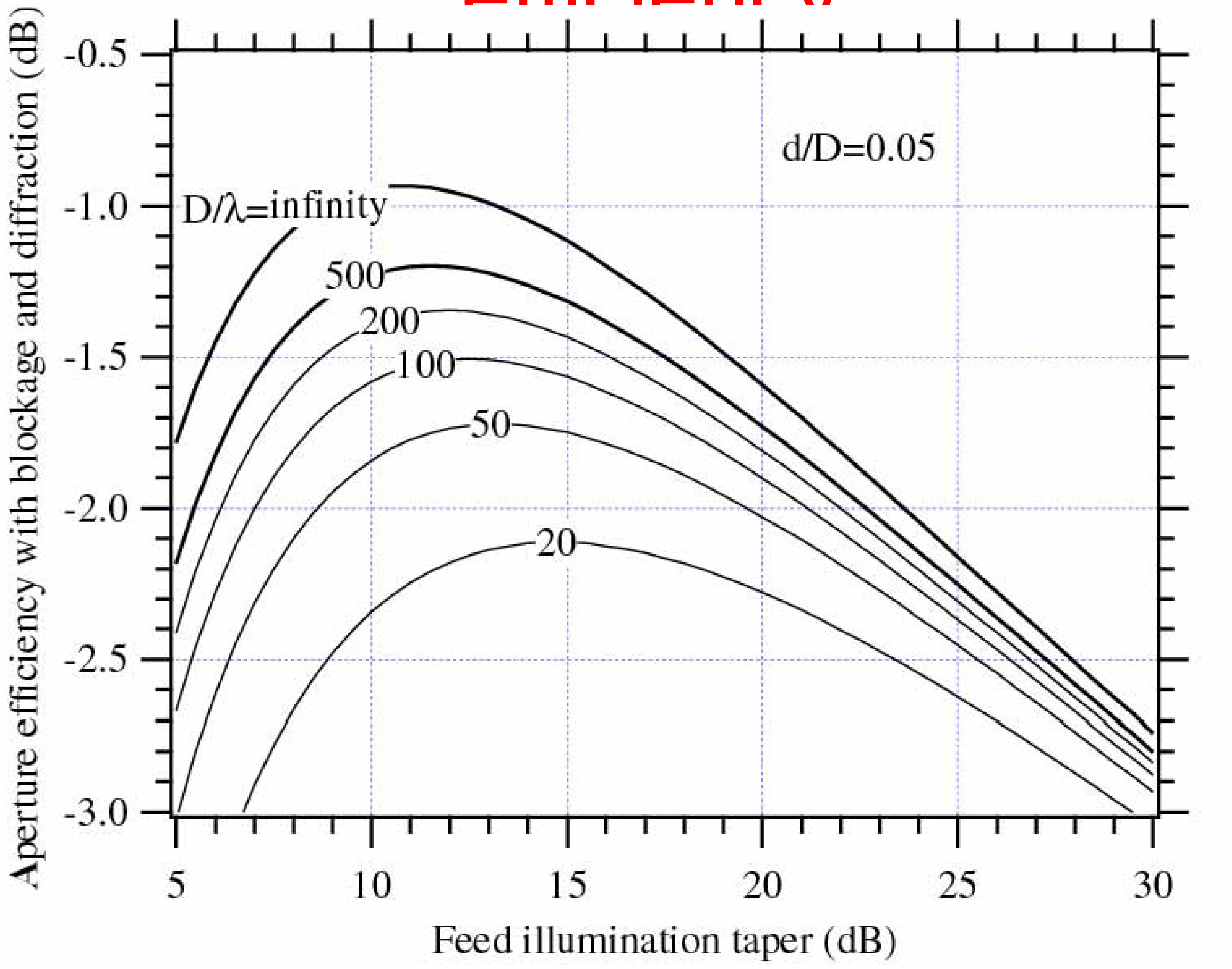
# Cassegrain: Subreflector diffraction and blockage

GO is only valid when reflectors are large.
Finite diameter of subreflector cause edge diffraction losses.
To keep losses small $d > 10\lambda / \sin(\psi_0)$



# Cassegrain: Total aperture efficiency



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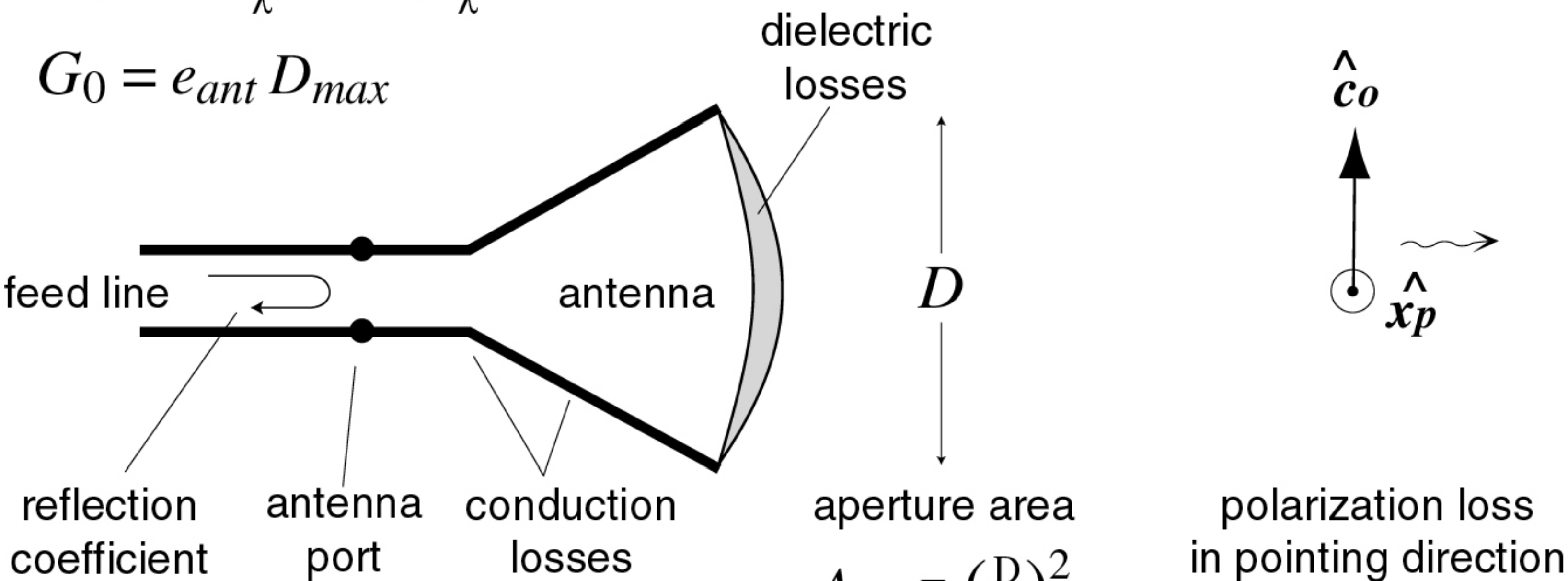
# System characteristics:
# Illustration of other subefficiencies

$$D_{max} = \frac{4\pi}{\lambda^2} A = \left(\frac{\pi D}{\lambda}\right)^2$$

$$G_0 = e_{ant} D_{max}$$



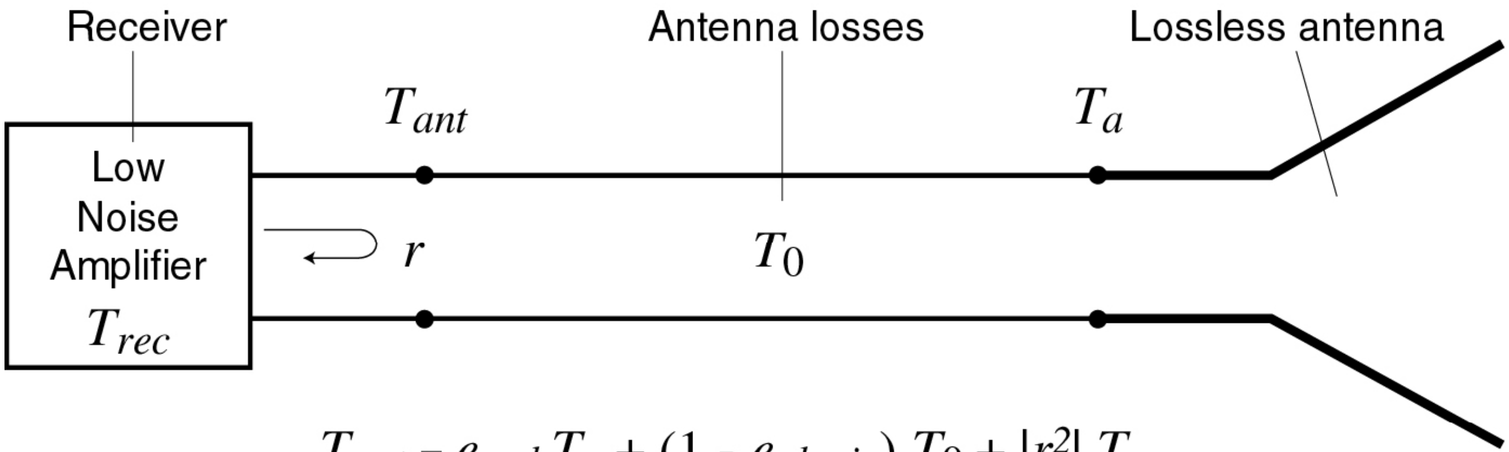

$$A = \pi \left(\frac{D}{2}\right)^2$$

$$e_{ant} = \underbrace{e_r \bullet e_{ohmic}}_{e_{rad}} \bullet e_{ap} \bullet e_{pol}$$

Will be replaced by reflector aperture efficiency in reflector antenna.

Effective area (receive): $A_{er} = \dfrac{\lambda^2}{4\pi} G_0$

# System performance: Figure of merit G/T




$$T_{ant} = e_{rad}\, T_a + (1 - e_{ohmic})\, T_0 + |r^2|\, T_{rec}$$

Figure of merit: $$\frac{G}{T_{syst}} = \frac{G}{T_{ant} + T_{rec}}$$

**G/T is measured in dB/K.**

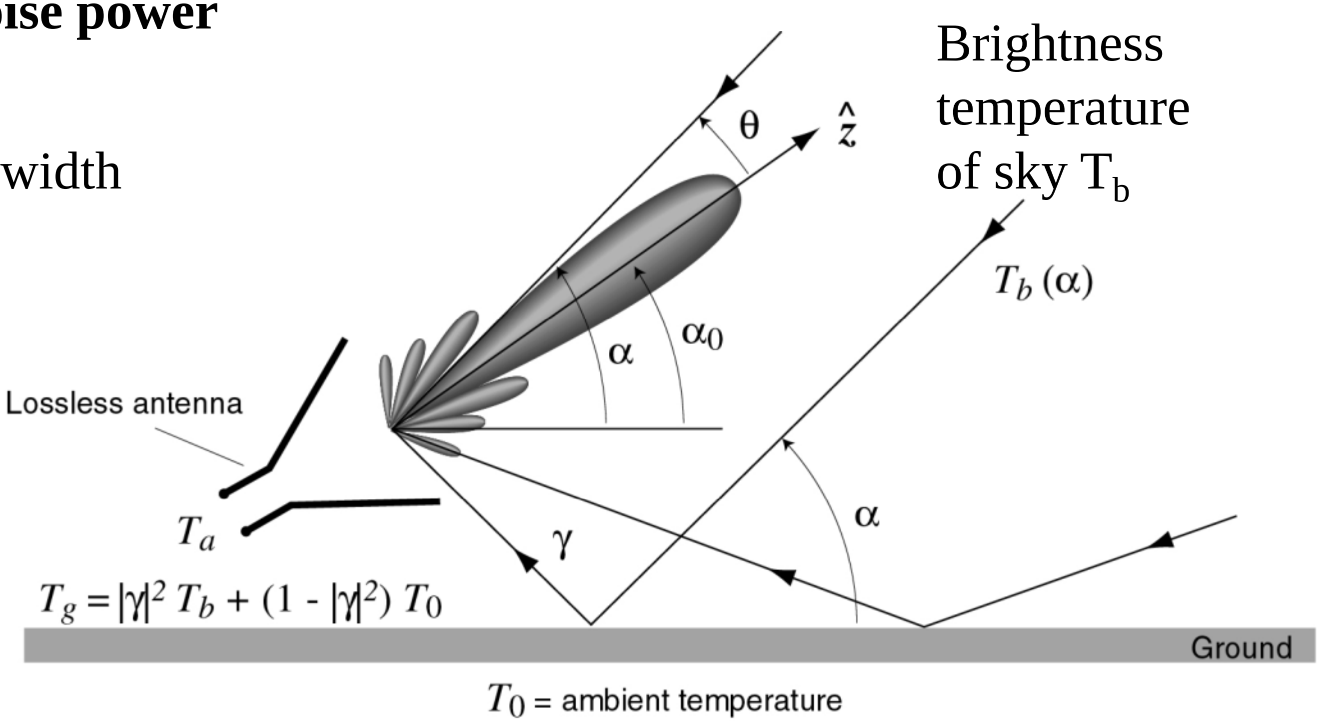
# System characteristics: Antenna temperature T

**T is a measure of the noise power P at the antenna port.**

bandwidth

$$P_{noise} = kT\Delta f$$

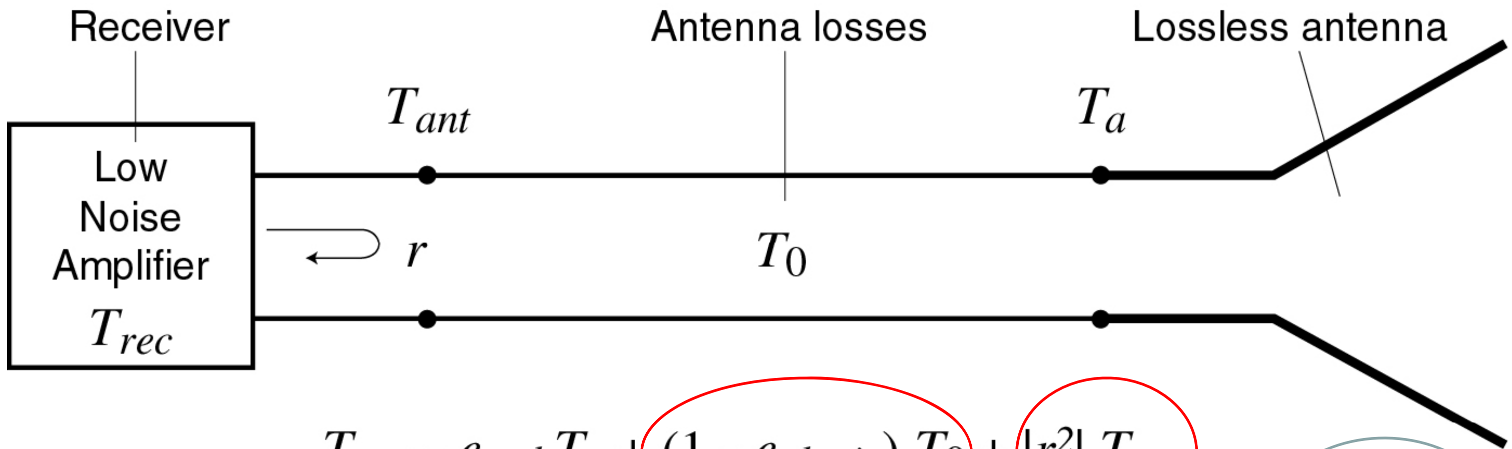
Bolzmann's const.




**Formula for effective antenna noise temperature (from sky & ground):**

$$T_a = \frac{\iint_{4\pi} (T_b + T_g)\, |G(\theta)|^2 \sin\theta \, d\theta \, d\phi}{\iint_{4\pi} |G(\theta)|^2 \sin\theta \, d\theta \, d\phi}$$

# System characteristics: Figure of merit G/T




$$T_{ant} = e_{rad}\,T_a + (1 - e_{ohmic})\,T_0 + |r^2|\,T_{rec}$$

Represents 7K per 0.1 dB loss

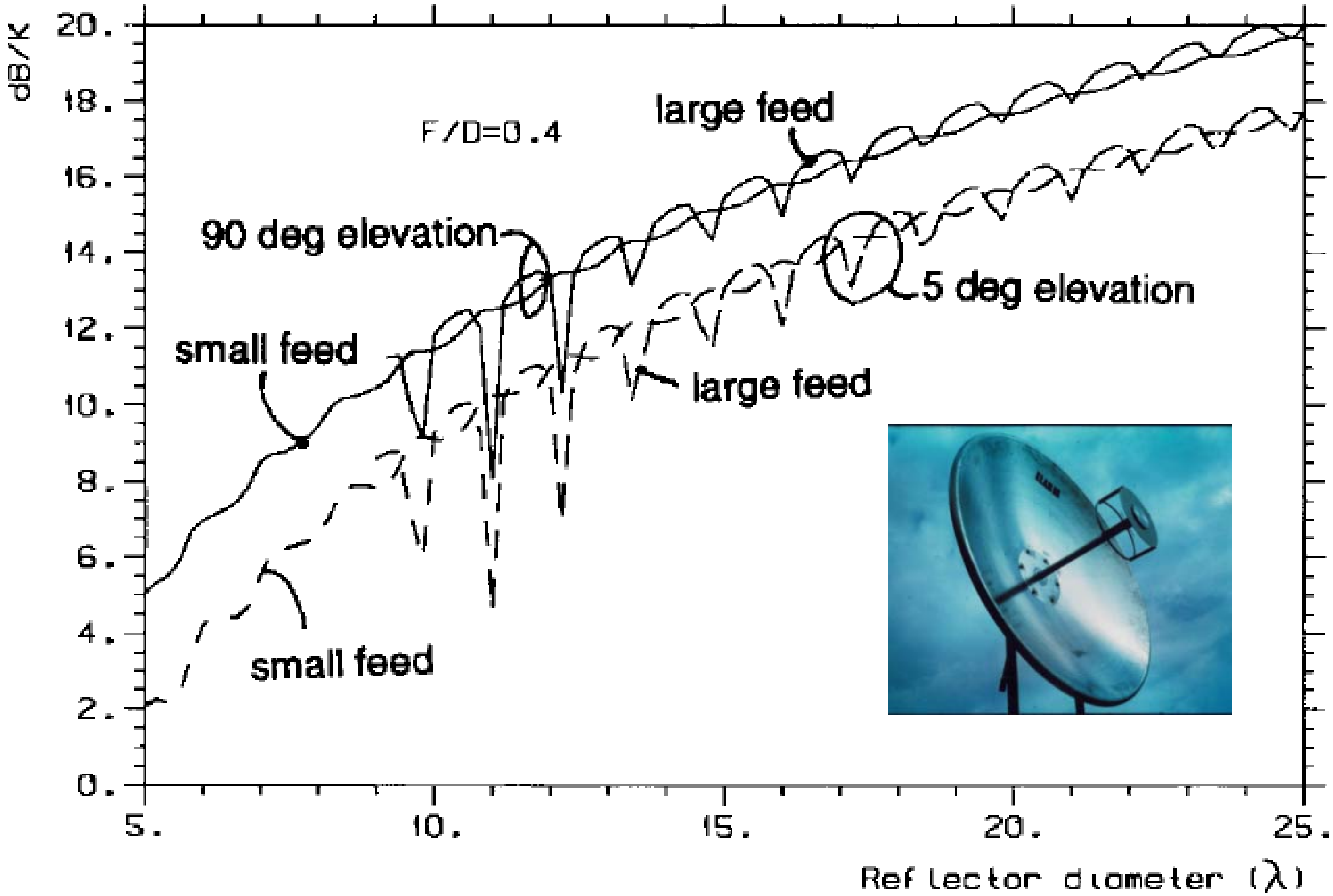
This is not completely correct

Figure of merit: $$\frac{G}{T_{syst}} = \frac{G}{T_{ant} + T_{rec}}$$

**The most common reference point of G and T are at the input of the LNA.**

**G/T is measured in dB/K.**

# G/T optimization of same small reflector with dipole-disk feed



# Surface tolerances

Finite surface accuracy causes phase errors

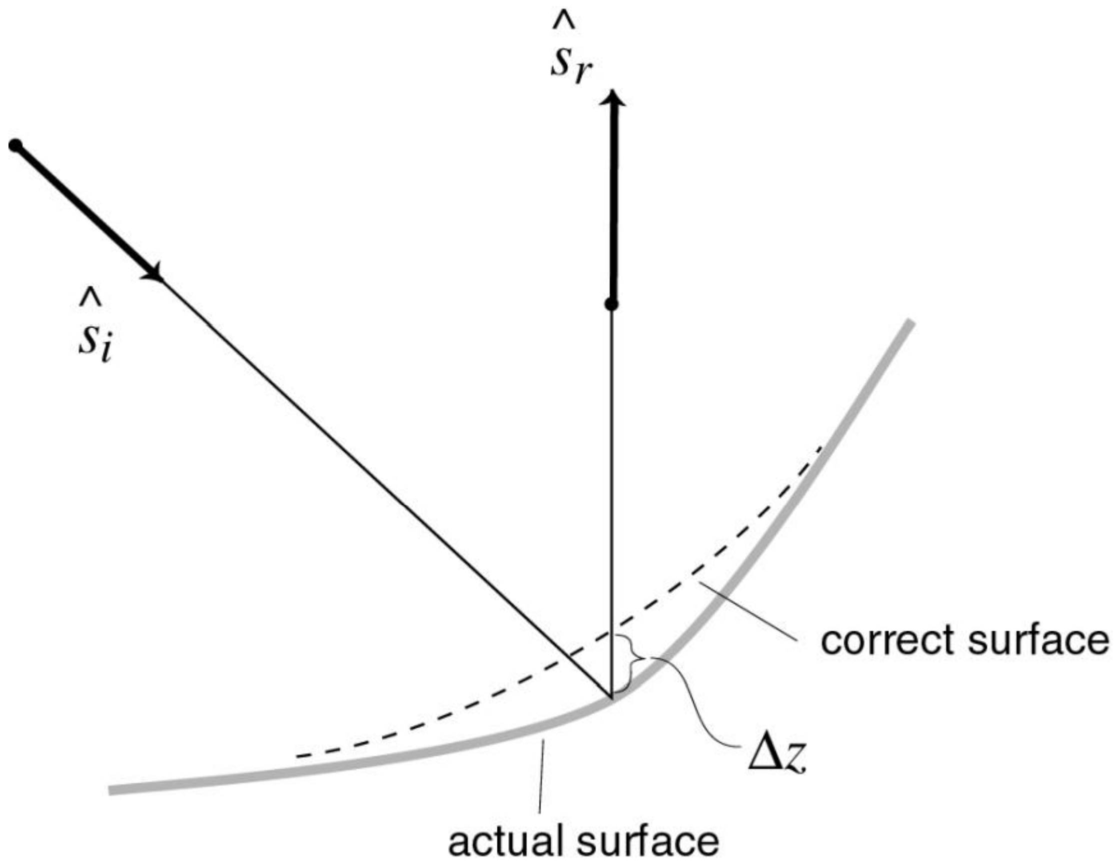
$$\phi(\theta, \varphi) = (1 + \cos\theta)k\Delta z(\theta, \varphi) \approx 2k\Delta z(\theta, \varphi)$$

Tolerance efficiency:

$$e_{tot} = 1 - (2k\Delta z_{rms})^2$$

For less than .3 dB loss we need

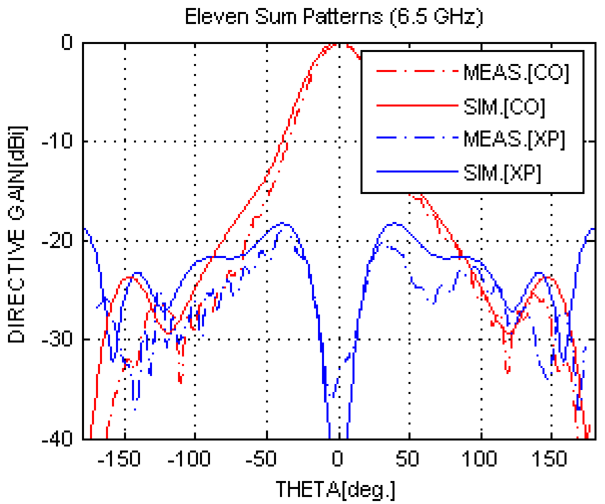
$$\Delta z_{rms} = \lambda / 50$$



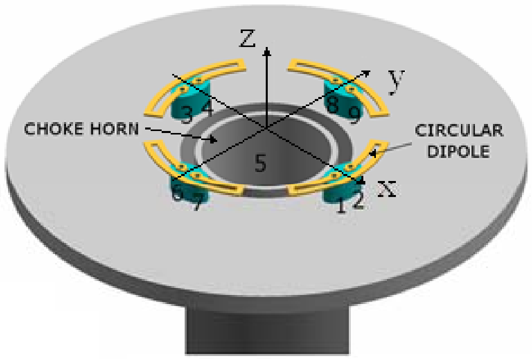
# Content of tutorial

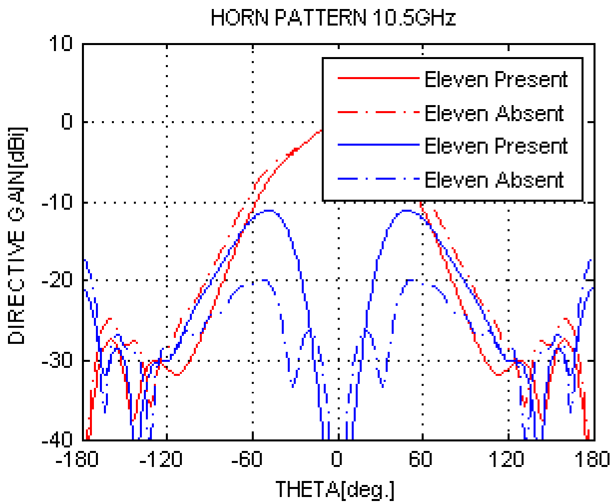
- Radiation field function (phase ref point)
- BOR antennas and BOR1 efficiency
- Subtended half angle versus F/D
- Feed efficiency and subefficiencies
  - Polarization, Spillover, Illumination, Phase (phase center)
- Blockage and subreflector diffraction
- More efficiencies
  - Mismatch factor, radiation efficiency
- How do they affect G/Tsys
- Combination feeds and decoupling efficiency

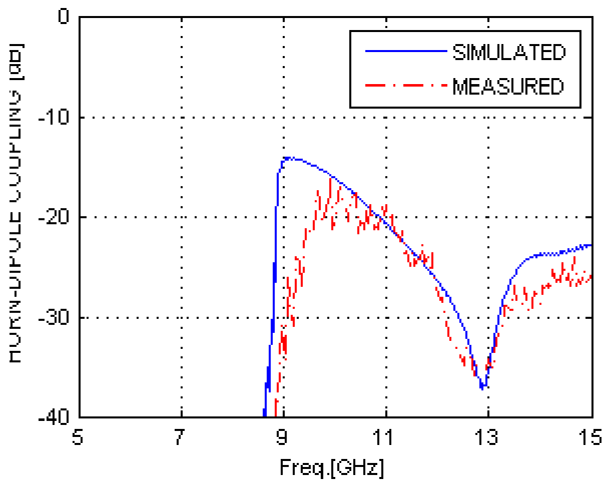
# Combination between choke horn and Eleven antenna

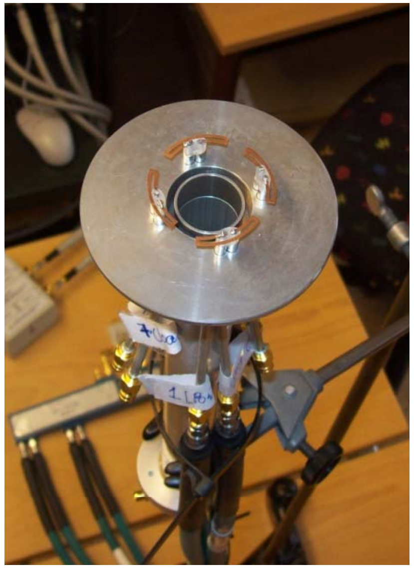



Minimum relative
frequency separation:
10.5/6.5 = 1.6



# Some important points

- BOR1 efficiency is important in non-BOR feed geometries
- In BOR1 feeds the co- and cross-polar radiation patterns determines ALL characteristics (phase center, illumination and spillover efficiency)
- Phase center can be uniquely defined from phase pattern in 45 deg plane and the phase efficiency
- Center blockage can give strong dips in G/T
- Airy’s disk type feed excitations are possible in Focal Plane Arrays, but not for single feeds
- Combination feeds (choke-horn and Eleven feed) are feasible for minimum relative frequency separation of 1.6

# Eleven feed