Absolute methods for determination of reference system from VLBI observations.

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Abstract:

There are two principal approaches to derivation of a reference system from various types of observations. Positions of stations and sources can be determined with respect to the reference frame taken as a fiducial one in differential approaches. Positions of a subset of some fiducial stations/sources which "hold" the system are fixed and are not estimated in the framework of this approach. The catalogues built by this way depend on the values of a priori positions of fiducial objects and therefore we call them "differential catalogues". However method of VLBI allows us to derive positions of all stations/sources from the observations up to a set of arbitrary constants. Since the reference system obtained by this way does not depend on the a priori catalogues it is called "absolute" reference system. Approaches for the development of such absolute catalogues of stations/sources using VLBI observations are discussed. Consequences on separation vertical component of station velocities from horizontal components are emphasized.

1 Introduction

The method of VLBI is purely geometric. Equation for an observed time delay may be written in the first approximation as

$$\tau = \frac{1}{c}\,\hat{\mathcal{E}}\,(\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2) \cdot \hat{\mathcal{H}}\,\vec{\mathbf{s}} \tag{1}$$

where $\hat{\mathcal{E}}$ — a rotation matrix which takes into account diurnal Earth rotation and pole motion, $\vec{\mathbf{r}}$ — a station position vector in geocentric coordinate system, $\hat{\mathcal{H}}$ – a rotation matrix which takes into account precession and nutation and $\vec{\mathbf{s}}$ — a unit vector of source position in baricentric reference system. Time delay is proportional to a projection of the baseline vector on the source vector. Having observed different sources at different baselines we are in principle able to get adjustments to the station positions, source coordinates and parameters of the matrices $\hat{\mathcal{E}}$ and $\hat{\mathcal{H}}$ using estimation technique like LSQ [1].

Two estimation strategies are possible. We can determine positions of some subset of stations and sources while leaving positions of other stations and sources fixed. This approach is called "differential". Estimates of positions of the targeted objects depend on the values of positions of the fixed objects. Errors in a priori values of positions of fixed objects as well as the errors in the Earth orientation parameters (EOP) if they are not estimated propagate to the estimates of positions of the targeted objects. We can use differential approach when the expected precision in determination of positions of targeted objects is less than the precision of positions of the fixed objects. In this case an additional error due to the uncertainty in positions of fixed objects does not bring substantial contribution to the general error budget. It is plausible to apply the differential method in estimation of positions of mobile VLBI stations.

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However using the differential approach we have to get a good input catalogue of positions of the objects which are to be used as fixed. VLBI nowadays is the most precise technique and we are unable to get EOP, station and source positions determined by other techniques without substantial degradation of precision. Thus, we have to have tools for estimation of positions and velocities of all stations, coordinates of all sources as well as EOP series from a single global solution. This approach we call "absolute".

2 Discussion

We encounter a specific difficulty in making a global solution. The design matrix of this problem turns out to be a matrix of incomplete rank and the normal matrix which corresponds to it be singular. The reason of it stems from the fact that the equation (1) is invariant with respect to a group of linear transformations and therefore the columns of the design matrix are linearly dependent.

- Time delay is invariant with respect to a translation of terrestrial reference system. If we add an arbitrary vector to all vectors of station positions, the time delay remains the same.
- Time delay is invariant with respect to a turn of terrestrial reference system. If we multiply all vectors of station positions by an arbitrary rotation matrix and multiply matrix $\hat{\mathcal{E}}$ by the matrix to be reciprocal to this matrix, the time delay remain the same.
- Time delay is invariant with respect to a translation of terrestrial velocities field. If we add an arbitrary vector to all vectors of station velocities, the time delay remains the same.
- Time delay is invariant with respect to a rotation of terrestrial reference system. If we multiply all vectors of station velocities by an arbitrary rotation matrix and multiply matrix $\hat{\dot{\mathcal{E}}}$ by the matrix to be reciprocal to this arbitrary matrix, the time delay remains the same.
- Time delay is invariant with respect to a turn of celestial reference system. If we multiply all vectors of source positions by an arbitrary rotation matrix and multiply matrix $\hat{\mathcal{H}}$ by the matrix to be reciprocal to this matrix, the time delay remain the same.

We have 5 vector invariance conditions, each of them depends on 3 parameters. Thus, the equation (1) is invariant with respect to a 15-parameters linear transformation when we determine positions and velocities of all stations, EOP series and positions of all sources.

Invariance conditions can be written as

$$
\left\{
\begin{array}{llll}
\sum\limits_i^m \Delta\vec{\mathbf{r}}_i & & & = \mathrm{const} \\
\sum\limits_i^m \dfrac{\vec{\mathbf{r}}_i \times \Delta\vec{\mathbf{r}}_i}{r_i^2} & & & = \mathrm{const} \\
\sum\limits_i^m \Delta\vec{\mathbf{v}}_i & & & = \mathrm{const} \\
\sum\limits_i^m \dfrac{\vec{\mathbf{r}}_i \times \Delta\vec{\mathbf{v}}_i}{r_i^2} & & & = \mathrm{const} \\
\sum\limits_j^n & \sin\alpha_j \cdot \Delta\delta_j & - \cos\delta_j \cos\alpha_j \sin\delta_j \cdot \Delta\alpha_j & = \mathrm{const} \\
\sum\limits_j^n & - \cos\alpha_j \cdot \Delta\delta_j & - \cos\delta_j \sin\alpha_j \sin\delta_j \cdot \Delta\alpha_j & = \mathrm{const} \\
\sum\limits_j^n & & \cos^2\delta_j \Delta\alpha_j & = \mathrm{const}
\end{array}
\right.
\tag{2}
$$

where m is the number of stations and n is the number of sources.

To solve equations of conditions of incomplete rank we can either use equations (2) as equations of constraints or to use conditional LSQ (i.e to find estimates of the parameters which provides minimum to a weighted sum of residuals under condition that equations (2) are exactly fulfilled, refer [2] for details). Adjustments depend on the choice of constants in (2) and implicitly depend on the values of a priori positions of stations and sources and EOP series. We can compute values of these constants by the manner which provide getting adjustments which would satisfy to explicitly specified boundary conditions. Another way is to get a solution using arbitrary values of these constants and then fulfill transformation of preliminary output catalogues and EOP series. The meaning of the solution with constants equal to zero ("zero solution") is that the output catalogues of station positions, station velocities, source positions don't have net translation net turn and net rotation.

It is worth noting that equations (2) may be written in alternative forms. In general, the matrix of the equations (2) can be multiplied by an arbitrary non-singular matrix. Implicit form of imposing some invariance conditions may be also used: for example, not estimation of some parameters.

The transformation of terrestrial and celestial reference systems, EOP series which conserves time delay (1) may be written in the form:

$$\begin{aligned} \vec{\mathbf{r}}_n &= \vec{\mathbf{r}}_o + \Delta\vec{\mathbf{r}}_e + \vec{\mathbf{T}} + \vec{\mathbf{\Phi}} \times \vec{\mathbf{r}}_o \\ \vec{\mathbf{v}}_n &= \vec{\mathbf{v}}_o + \Delta\vec{\mathbf{v}}_e + \vec{\mathbf{V}} + \vec{\mathbf{\Omega}} \times \vec{\mathbf{r}}_o \\ \vec{\mathbf{s}}_n &= \vec{\mathbf{s}}_o + \Delta\vec{\mathbf{s}}_e + \vec{\mathbf{\Psi}} \times \vec{\mathbf{s}}_o \end{aligned} \tag{3}$$

where $\vec{\mathbf{r}}_n$ — a vector of station position in a new reference system, $\vec{\mathbf{r}}_o$ — a vector of station position in an old reference system, $\Delta\vec{\mathbf{r}}_e$ — a vector of adjustments to the station position obtained from the solution, $\vec{\mathbf{T}}$ — a translation vector of terrestrial reference system, $\vec{\mathbf{\Phi}}$ — a small turn vector of terrestrial reference system. Analogously, $\vec{\mathbf{V}}$ and $\vec{\mathbf{\Omega}}$ are a translation and a small turn vectors of station velocities field and $\vec{\mathbf{\Psi}}$ is a small turn vector of celestial reference system.

EOP series are transformed by the following way:

$$\begin{aligned} \mathrm{Yp}_n &= \mathrm{Yp}_o + \Delta\mathrm{Yp}_e + \vec{\mathbf{\Phi}}_1 + \vec{\mathbf{\Omega}}_1 \cdot (t - t_o) \\ \mathrm{Xp}_n &= \mathrm{Xp}_o + \Delta\mathrm{Xp}_e + \vec{\mathbf{\Phi}}_2 + \vec{\mathbf{\Omega}}_2 \cdot (t - t_o) \\ \mathrm{UT1}_n &= \mathrm{UT1}_o + \Delta\mathrm{UT1}_e - k(\vec{\mathbf{\Phi}}_3 - \vec{\mathbf{\Psi}}_3) - k\vec{\mathbf{\Omega}}_3 \cdot (t - t_o) \\ \frac{\partial}{\partial t}\mathrm{UT1}_n &= \frac{\partial}{\partial t}\mathrm{UT1}_o + \Delta\frac{\partial}{\partial t}\mathrm{UT1}_e - k\vec{\mathbf{\Omega}}_3 \\ \varepsilon_n &= \varepsilon_o + \Delta\varepsilon_e - \vec{\mathbf{\Psi}}_1 \\ \psi_n &= \psi_o + \Delta\psi_e + \frac{1}{\sin\varepsilon_0}\vec{\mathbf{\Psi}}_2 \end{aligned} \tag{4}$$

where t_o is the epoch of station coordinates, $k = 1.002\,737\,909$ and ε_0 is a mean obliquity of ecliptic to equator.

As it was previously mentioned equation (1) is an approximation. Time delay is computed with a relative precision 10^{-11} on the basis of a refined model [3]. The two major omitted terms are

$$\frac{1}{c^2}\left(\hat{\mathcal{E}}\,(\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2) \cdot \vec{\mathbf{V}}_\oplus\right) - \frac{1}{c^2}\left(\hat{\mathcal{E}}\,(\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2) \cdot \hat{\mathcal{H}}\,\vec{\mathbf{s}}\right)\left(\vec{\mathbf{V}}_\oplus \cdot \vec{\mathbf{s}}\right) \tag{5}$$

where $\vec{\mathbf{V}}_\oplus$ is baricentric Earth velocity.

The vector of baricentric Earth velocity usually is not an estimated parameter and therefore these terms are not invariant with respect to a turn of celestial reference frame. Baricentric Earth velocity is taken from ephemerides which provide it with a precision rather better than it can be determined from VLBI observations. Ephemerides keep the orientation of the dynamic reference frame in which they are defined and are not undergone transformation. Therefore, applying transformation (3) and (4) may cause an error. We can neglect this error provided the vector $\vec{\mathbf{\Psi}}$ is small enough:

$$\frac{\|\sigma(\vec{\mathbf{s}})\|}{\|\vec{\mathbf{\Psi}}\|} \ll \frac{\|\vec{\mathbf{V}}_\oplus\|}{c} \tag{6}$$

where c is light velocity, $\sigma(\vec{\mathbf{s}})$ — an uncertainty of source vector adjustments. Since $\dfrac{\|\vec{\mathbf{V}}_{\oplus}\|}{c} \approx 10^{-4}$ this condition is usually fulfilled. Influence of other omitted terms is less and can be wholly ignored.

We have a freedom of choice of $\vec{\mathbf{T}}$, $\vec{\mathbf{V}}$, $\vec{\mathbf{\Phi}}$, $\vec{\mathbf{\Omega}}$, $\vec{\mathbf{\Psi}}$. Freedom in choice of translation vector $\vec{\mathbf{T}}$ leads to the fact that separation vertical and horizontal components in vectors of station positions is up to some degree arbitrary. Let we have 6 stations as it seen in fig. 1. Having added a translation vector $\vec{\mathbf{T}}$ we change positions of stations so that the stations a and b are uplifting, the stations e and d are sinking and no additional vertical motion is added to the stations c and d. If we change direction of the vector $\vec{\mathbf{T}}$ at 90° as it shown in fig. 2 then the station c will be uplifting, d sinking and vertical positions of the stations a, b, d and e remain the same. Applying a turn of the reference system makes the whole picture more complicated due to an additional horizontal displacement caused by the turn of reference system which depends on the relative positions of the station coordinates vectors and the vector $\vec{\mathbf{\Phi}}$ (fig 3). Applying different vectors $\vec{\mathbf{T}}$ and $\vec{\mathbf{\Phi}}$ we get different vertical positions of the stations. Analogously, applying different vectors $\vec{\mathbf{V}}$ and $\vec{\mathbf{\Omega}}$ we get different values of vertical and horizontal velocities of the stations. One choice $\vec{\mathbf{V}}$ and $\vec{\mathbf{\Omega}}$ may reveal an uplift of the targeted station while another choice of $\vec{\mathbf{V}}$ and $\vec{\mathbf{\Omega}}$ may reveal sinking the same station. And both solutions satisfy observations!

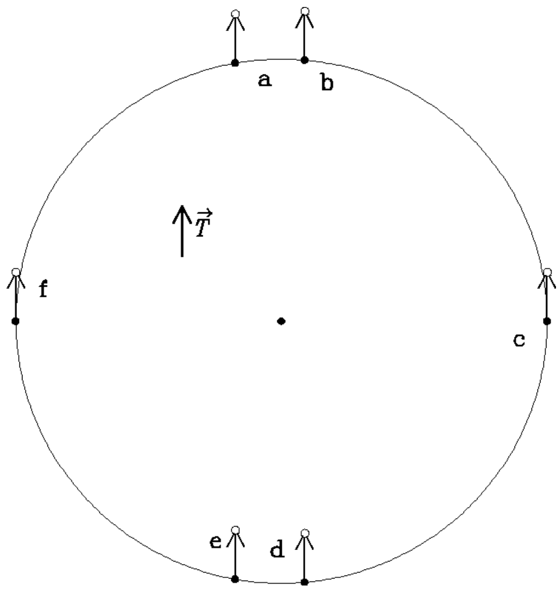

fig. 1

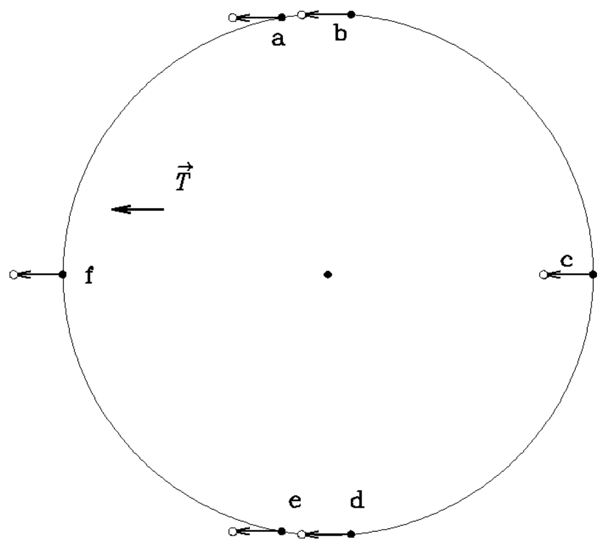

fig. 2

I would like to emphasize that we are in principle unable to determine parameters of this transformation from VLBI observations. Therefore one possible way is not to worry about some arbitrariness of the output solutions. However a raw solution *implicitly* depends on used equations of invariance conditions and a priori catalogues. Changes in a priori catalogues, changes in equations of invariance conditions, addition or removal some observations from analysis may lead to considerable changes in values of the output coordinates and EOP series what may cause an illusion of "instability" of solution. In order to avoid these problems we can transform our raw solution on the basis of *explicitly* specified boundary conditions using external information.

Several strategies for obtaining parameters of the transformation can be used.

— Vectors of the transformation can be usually chosen to make a new solution compliant with catalogues of stations and sources and EOP series determined by other techniques. We explicitly formulate boundary conditions of compliance. These conditions set equality of some coordinates of stations, sources and/or EOP series to the values from the external catalogues. As an example

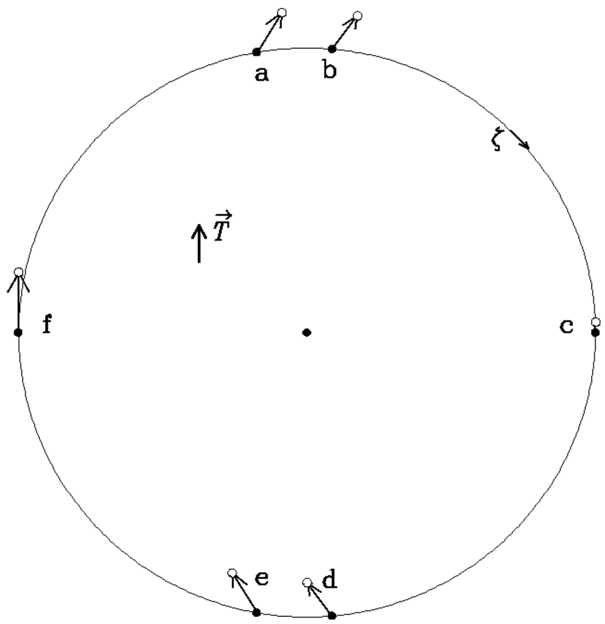

fig. 3

we can demand equality of final EOP to the values from USNO concrete EOP series on the specific date.

— We can modify boundary condition and to require not rigorous equality of the values but minimization of a norm of differences of the new catalogues and the old external catalogues. For example, we can demand that a sum of squares of differences between the new sources catalogue and the IERS sources catalogue would be minimal.

— Boundary conditions can be set for compliance with some physical considerations. For example, we can require that our new field of velocities would not have rotation with respect to some model of plate tectonic or we can require that some subset of stations has minimal deviation from the model of post-glacial rebound or we can require that the pole coordinates don't have secular trend.

Boundary conditions set a system of linear equations which ties parameters of the transformation (3), (4), external catalogues or model with zero solution. Having solved these equations we find parameters of the transformation. After applying this transformation we get the final catalogues and EOP series.

3 Conclusion

The VLBI output catalogues and EOP series contains both information which is *derived from the measurements* and external information which is *imposed.* The fact that terrestrial reference frame and velocities field can be determined only up to transformation (3), (4) puts some restrictions on interpretation of the results obtained from VLBI.

- Earth Orientation parameters and positions of all stations cannot be unambiguously determined from a single 24 hours VLBI experiment. However baseline lengths are invariant with respect to translation and rotation and can be determined without applying external conditions.

- Secular trend in Earth orientation parameters series cannot be in principle measured by VLBI.

- If we analyze VLBI data only from the region small with respect to the Earth radius we can determine vertical and horizontal component of station positions and station velocities only up to a common arbitrary vector. But we still can determine relative vertical and horizontal position and velocities.
- If we compare two different VLBI solutions they should be first undergone a transformation: rotation and translation to fit the same boundary conditions.

References

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