Steps towards phase delay VLBI

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Abstract:

One of the promising ways for improving precision of estimation of the targeted parameters of geodetic VLBI is to use phase delay measurements which are roughly 40-fold more precise than group delay observables. However phase delay ambiguities should be resolved before using these observables in LSQ adjustments. This problem is solved almost always for the case of relatively short baselines (1–10 km) where we are able to ignore an atmosphere contribution to time delay, but it is still a challenge for the case of longer baselines.

A procedure for resolving phase delay ambiguity is presented. It exploits the fact that phase misclosures are usually a small fraction of phase ambiguity spacings. Phase delay measurements made in the network consisting of three or more stations provide some redundancy.

It is shown that the proposed approach allows to resolve phase delay ambiguities for a subset of observations. Some results of phase delay solutions of EUROPE, IRIS-S and VLBA networks are presented. It was found that the success rate of phase delay ambiguity resolution for intercontinental baselines depends on the system sensitivity, the level of instrumental noise and the number of observations. Phase delay ambiguities can be resolved for up to 90% observations for the best, state-of-art VLBI experiments, although a success rate 40-70% is more typical.

1 Introduction

Phase spectrum of cross-correlation function within a band of registration is produced by a correlator in processing very long baseline interferometry measurements [1]. The correlator output is used for

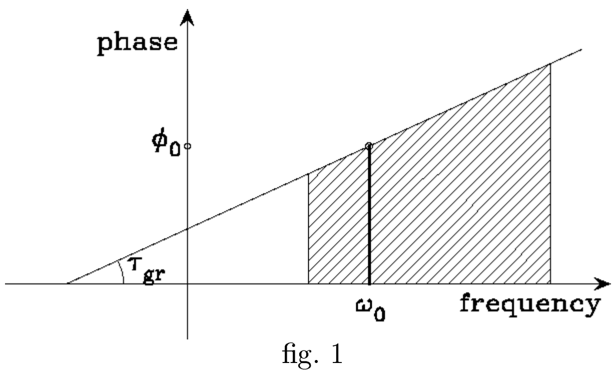

fig. 1

obtaining two basic quantities: group delay and phase delay:

$$\tau_{group} = \frac{\partial \varphi}{\partial \omega} \qquad \tau_{phase} = \frac{\varphi}{\omega_o} + \frac{2\pi}{\omega_o} N \tag{1}$$

where φ is a phase of cross-spectrum, ω_o is a circular reference frequency and N — an integer number of ambiguities. Precision of measurement of these quantities is:

$$\sigma(\tau_{group}) \sim \frac{1}{\mathrm{SNR} \cdot \Delta\omega} \qquad \sigma(\tau_{phase}) \sim \frac{1}{\mathrm{SNR} \cdot \omega_o} \tag{2}$$

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where SNR is a signal to noise ratio of fringe detection, $\Delta\omega$ — effective bandwidth. The bandwidth of registration is considerably less than the reference frequency and therefore the precision of measurement of phase delays is much higher than the precision of measurements of group delays. But there is a substantial obstacle before direct use of phase delay: it is measured only up to an unknown integer number of phase turns. We are able to use phase delays in a parameter adjustment process only after resolving phase delay ambiguities. Early experiments showed [2], [3] that the phase delays may be successfully used for analysis of observations made at very short baselines of 0.1–1.0 km where phase delay ambiguities are resolved rather easy. Now we try to resolve phase delay ambiguities for baselines of arbitrary length.

2 Phase delay observables

Group and phase delays at X ($\sim$ 8.4GHz) and S ($\sim$ 2.3GHz) bands are measured in geodetic VLBI observations. Properties of these observables are presented in table 1.

Tab. 1

	Group delay	**Phase delay**
Precision	X–band: 0.02–0.04 nsec S–band: 0.01–0.2 nsec	X–band: 0.0005–0.001 nsec S–band: 0.002–0.005 nsec
Ionosphere contribution	X–band: 0.2–2.0 nsec S–band: 2.5–30.0 nsec	X–band: 0.2–2.0 nsec S–band: 2.5–30.0 nsec
Ambiguity	X–band: 28–100 nsec S–band: 50–200 nsec	X–band: 0.12 nsec S–band: 0.45 nsec
postfit rms	0.02–0.03 nsec	0.005–0.010 nsec

We see that a scatter of post-fit residuals of group delays has a magnitude 2–5 times less than phase delay ambiguity spacings at X-band and 5-20 times less than ambiguity spacings at S-band. In principle, phase delay ambiguities can be resolved by using model values of phase delay derived from a group delay solution provided the following conditions are true:

- Theoretical model for group and phase delay differs by no more than 0.2–0.3 phase turns;
- Instrumental difference between group and phase delay does not exceed 0.2–0.3 phase turns;

However, both these conditions are usually not fulfilled. Three contributions to time delay are different for phase and group delays: feed horn rotation, source structure correction and ionosphere contribution. The first term can be precisely calculated. Difference in source structure correction is usually less than 0.1 phase turns. Unfortunately, ionosphere contribution is substantially different for group and for phase delay observables what puts serious problems before a phase delay ambiguity resolution.

Ionosphere contribution to time delay can be written in the form

$$\tau^{iono}_{group} = \frac{q}{f_g^2} \qquad \tau^{iono}_{phase} = -\frac{q}{f_p^2} \tag{3}$$

where q is a quantity proportional to the difference of integral electron content over the ray paths from the source to the site 1 and the site 2, f_g and f_p — effective frequencies for group and phase delays which are within 3–5% from the reference frequency. Difference in the ionosphere contribution as it is seen in table 1 is about 3–15 phase turns and we unable to ignore this effect in a phase delay ambiguity resolution process.

To take into account the ionosphere contribution we use ionosphere free linear combinations of observables. The most important combinations are:

$$
\begin{aligned}
\tau_g^{if} &= \frac{f_{gx}^2}{f_{gx}^2 - f_{gs}^2}\tau_{gx} - \frac{f_{gs}^2}{f_{gx}^2 - f_{gs}^2}\tau_{gs} \\
\tau_p^{if} &= \frac{f_{px}^2}{f_{px}^2 - f_{ps}^2}\tau_{px} - \frac{f_{ps}^2}{f_{px}^2 - f_{ps}^2}\tau_{ps} \\
\tau_{pxgs}^{if} &= \frac{f_{px}^2}{f_{px}^2 + f_{gs}^2}\tau_{px} + \frac{f_{gs}^2}{f_{px}^2 + f_{gs}^2}\tau_{gs} \\
\tau_{psgs}^{if} &= \frac{f_{ps}^2}{f_{ps}^2 + f_{gs}^2}\tau_{ps} + \frac{f_{gs}^2}{f_{ps}^2 + f_{gs}^2}\tau_{gs}
\end{aligned}
\tag{4}
$$

where f_{gx}, f_{gs} are effective frequencies for group delay ionosphere contribution at the bands X and S and f_{px}, f_{ps} are effective frequencies for phase ionosphere contribution. Having substituted typical values of the effective frequencies we can write (4) in this form:

$$
\begin{aligned}
\tau_g^{if} &\approx 1.08\tau_{gx} - 0.08\tau_{gs} \\
\tau_p^{if} &\approx 1.08\tau_{px} - 0.08\tau_{ps} \\
\tau_{pxgs}^{if} &\approx 0.94\tau_{px} + 0.06\tau_{gs} \\
\tau_{psgs}^{if} &\approx 0.51\tau_{ps} + 0.49\tau_{gs}
\end{aligned}
\tag{5}
$$

where index p, or g denotes phase or group delays, x and s denotes a band. The first ionosphere free linear combination is used in group delay solutions. The second combination is called pure phase delay ionosphere free linear combination and the latter two are called semi-phase delay ionosphere free linear combinations.

3 Phase delay ambiguity resolution

In general, the problem of phase delay resolution has more than one solution. To provide a unique solution we have to postulate some additional conditions. The following principles are formulated:

- *Group delay ionosphere free linear combinations and phase delay ionosphere free linear combinations don't have considerable differences.* In the absence of source structure contribution and instrumental errors these two quantities should coincide. Thus, this assumption is equivalent to the assumption that instrumental noise is small.

- *Misclosures in phase delays are considerably less than phase delay ambiguity spacings.* It was found that misclosures in phases are at the level of 0.05–0.15 phase turns for the sources to be observed under geodetic programs. It means that the instrumental noise is primarily station-dependent but not baseline-dependent.

- *There exist a smooth function of a difference between group delay ionosphere free linear combination and ionosphere free combination of phase delays.* We assume that a random component in differences is small. We have to somehow model this difference and the number of parameters of such a model should be small enough to secure uniqueness of the solution. We are unable to resolve correctly phase delay ambiguities if we allow considerable fluctuations of the differences. We cannot say beforehand whether this assumption is correct or not. When this condition is not valid then a phase delay ambiguity resolution process fails and we have to exclude such baselines from the list of baselines eligible for phase delay solutions.

Method of ambiguity resolution functions (ARF) is proposed to resolve phase delay ambiguities. The following baseline-dependent quantities are introduced:

$$\begin{aligned} X^{pa}_{ARF} &= -(t^{if}_{pxgs} - (\tau_c + \tau_a)) \cdot \frac{2\pi}{\omega_o} \\ S^{pa}_{ARF} &= -(t^{if}_{psgs} - (\tau_c + \tau_a)) \cdot \frac{2\pi}{\omega_o} \end{aligned} \qquad (6)$$

where, t^{if}_{pxgs}, t^{if}_{psgs} are ionosphere free linear combinations of group and phase delay observables from (4), τ_c — theoretical ionosphere free time delay, τ_a — contribution to time delay from adjustments obtained in the previous solution. Analogously, a station-dependent ARF can be introduced, so that the ARF for the specified baseline is a difference of the ARF for the stations forming the baseline. Values of station-dependent ARF are computed for each scan by solving equations which tie ARF with observed phase and group delays. ARF for one station among the stations participated in this scan is not determined and is considered as a reference one. Usage of station-dependent ARF instead of baseline-dependent ARF brings considerable merit if more than two stations participated in the observations of the scan. Each scan has $\frac{n(n-1)}{2}$ baseline-dependent ambiguities, but only $n-1$ are independent since they should satisfy to a closure conservation condition for each triplet of stations:

$$\frac{1}{2\pi}(\varphi_{12} + \varphi_{23} - \varphi_{13}) - \frac{\dot{\tau}_{23}\tau_{12}}{\omega_o} + N_{12} + N_{23} - N_{13} = 0 \qquad (7)$$

ARF would have only integer values in the absence of instrumental errors. Phase delay ambiguity resolution is reduced in this case to elimination of an integer part from ARF and redistribution of baseline-dependent phase delay ambiguities. Instrumental errors which are intrinsic to phase delays enter ARF directly. Instrumental errors which are intrinsic only to the group delays enter ARF directly and indirectly via bias in τ_a which is obtained in group delay solution. Thus, ARF is contaminated by both integer phase delay ambiguities and instrumental noise of measurement of group and phase delays in realistic case. Raw ARF is undergone by a filtering process. It is modeled as a sum of three components:

$$\mathrm{ARF}_i = r_i + h_i + N_i \qquad (8)$$

where, r_i — is a regular slowly variating smooth function, h_i — is a noise-like high frequency variating quantity and N_i — is a discontinuous ambiguity function which can have only integer values.

The regular component is modeled by a linear spline with imposing constraints on its rate of change. Parameters of the filter: duration of the segments (usually 1–2 hours) and the level of constraints (reciprocal weights are usually in the range 0.01–0.1 1/hour) control behavior of the regular component. The high frequency component is restricted by imposing an upper limit (usually 0.2–0.3) for its maximal in modulo value. Then station dependent phase delay ambiguities are determined as an integer part of the difference of the ARF value and the regular component. If a remnant in ARF after subtraction the regular part and the integer ambiguity exceeds the upper limit for the high-frequency component then that value of ARF is treated as an outlier.

Algorithm of phase delay ambiguity resolution can be described as follows:

1. Obtain the best group delay solution.
2. Form phase delays from measured wide-band fringe phases by applying a feed horn rotation correction. Obtain preliminary phase delay ambiguities from baseline-dependent ARF. Eliminate ambiguities in misclosures.
3. Compute post-adjustment station-dependent X-band ARF.
4. Filter station-dependent ARF with resolving station-dependent ambiguities;

5. Compute baseline-dependent ambiguities using station-dependent ambiguities;
6. Refine a PxGs (X-band semi-phase) delay solution by
   (a) Consecutive elimination of outliers from the solution up to the specified limit;
   (b) Backward restoration of the previously suppressed observations up to the specified limit;

   Errors made in phase delay ambiguity resolution during filtering ARF can be corrected in refinement of the initial solution provided the error rate was not high (no more than 10–20%).
7. Compute post-adjustment station-dependent S-band ARF;
8. Repeat steps 4, 5, then refine PsGs delay solution;
9. Produce PxPs combinations and refine pure phase delay solution;

The process of phase delay ambiguity resolution works satisfactory if the following assumptions are correct:

— Short periodic fluctuations of ARF (periods $< 3^h$) are less than 0.3 phase turn;

— Low frequency ARF rate is less than 0.3 turn/hour;

— Baseline-dependent systematic errors are negligible;

4 Results

Typical ARF before and after filtering are presented in the fig. 2–5. Argument in plots is time in hours elapsed since beginning of the experiment.

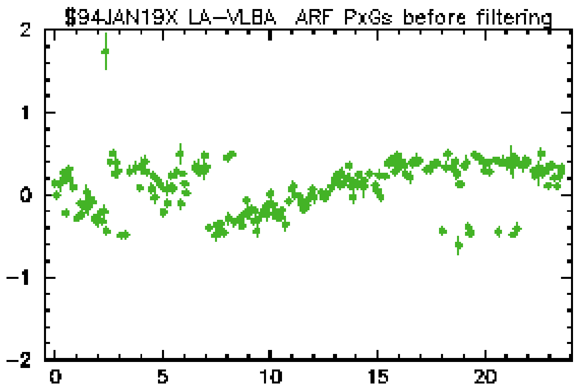

fig. 2

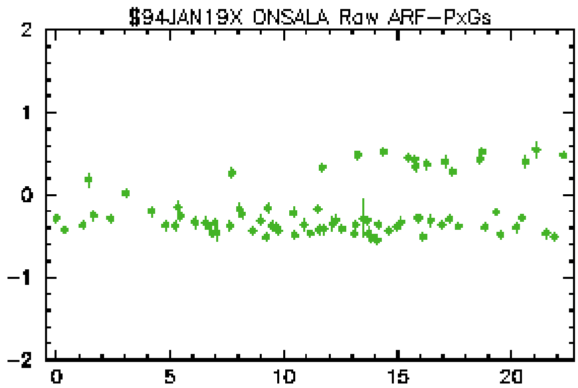

fig. 3

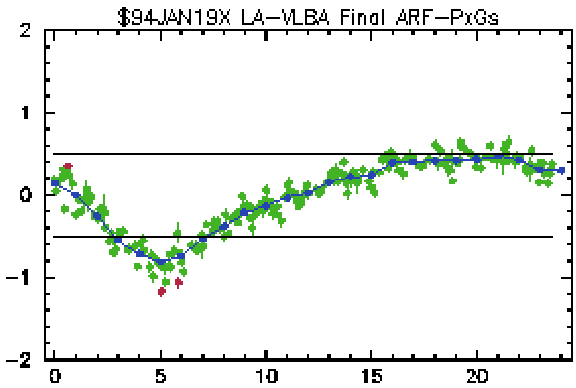

fig. 4

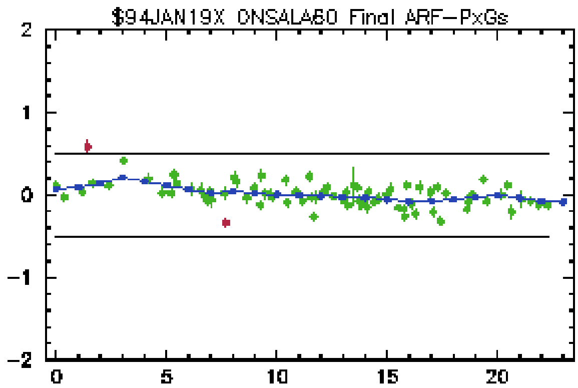

fig. 5

Solid line in fig. 4–5 represents adjustments of the regular component of ARF.

About 100 sessions were processed by algorithms of phase delay ambiguity resolution: experiments carried under programs EUROPE, IRIS-S, geo-VLBA, NEOS-A, NAVEX. I found that

- Phase delay ambiguities can be resolved for the baselines up to 10 thousand km. It is difficult to resolve ambiguity for the longer baselines since there are too few common observations at the baselines with length more than 85% of the Earth diameter.
- The most important factors which influence on the possibility to resolve phase delay ambiguities are station sensitivity and the level of instrumental noise.
- Phase delay ambiguities can be resolved for up to 80–90% of observations in geo-VLBA and R&D experiments, 60–80% of IRIS-S experiments and 40-70% of EUROPE and NEOS-A experiments.
- It is more difficult to resolve phase delay ambiguities at S-band than at X-band.
- There are stations which are not eligible for phase delay ambiguity resolution.

After resolving phase delay ambiguities we are able to use several types of linear combinations of phase and group observables. Properties of different types of solutions are presented in the table 2.

Tab. 2

Group delay combination			
Used combination:	$1.08\tau_{gx}$	$-0.08\tau_{gs}$	$+\tau_n$
Uncertainty (VLBA mode)	15ps	6ps	12ps
Uncertainty (best Mark-III)	25ps	10ps	15ps
Ambiguity spacings:	>15ns		
X-band semi-phase delay combination			
Used combination:	$0.94\tau_{px}$	$+0.06\tau_{gs}$	$+\tau_n$
Uncertainty (VLBA mode)	0.5ps	4ps	8ps
Uncertainty (best Mark-III)	1.0ps	8ps	8ps
Ambiguity spacings:	114ps		
Phase delay solution			
Used combination:	$1.08\tau_{px}$	$-0.08\tau_{ps}$	$+\tau_n$
Uncertainty (VLBA mode)	0.5ps	0.2ps	8ps
Uncertainty (best Mark-III)	1.0ps	0.3ps	8ps
Ambiguity spacings:	131ps and 34ps		

It is remarkable that there is a noise floor τ_n. Existence of this floor makes pure phase delay solutions not so attractive. We see that the advantages of a solution made by using a combination of phase and group delay observables called semi-phase with respect to a group delay solution are substantial. At the same time a pure phase delay solution brings only marginal improvement with respect to a semi-phase delay solution due to existence of the noise floor. Moreover, the pure phase delay solution is less reliable than the semi-phase delay solution, since it has two ambiguity spacings: 131 psec and 34 psec while the semi-phase delay solution has the only ambiguity spacing 114 psec. It is difficult to detect observations with wrongly resolved ambiguities with spacings 34 psec when the noise floor is only 4 times less, while it is rather more easy to guarantee a correctness of a phase delay ambiguity resolution when the ambiguity spacing is 15 times wider than the noise floor.

A continuous set of 22 sessions carried out under program CONT94 at the VLBA and NAVEX networks was analyzed. 13 stations participated. Baseline lengths were in the range 0.6–10.4 thousands km. 60 315 out of 66 857 observations were used in the group delay solutions. Weighted sum of root mean squares of postfit residuals was 18 psec. 53 049 phase delay observables were used in the semi-phase delay solutions. Weighted sum of root mean squares of postfit residuals was 8.1 psec. Precision of determination of geodynamical parameters improved by the factor 2. At the same time increasing precision didn't result in improving accuracy.

Baseline length repeatability is presented in the table 3. L is the baseline length. Differences are

Tab. 3

Phase delay solutions:	R(L) = 0.11 mm	+	$9.6 \cdot 10^{-10}$	$\times$ L
Group delay solutions:	R(L) = 0.19 mm	+	$8.7 \cdot 10^{-10}$	$\times$ L

not statistically significant. Detailed analysis of a baselines evolution reveals substantial variations in baseline lengths of unknown origin which affects both phase and group delay solutions. Plots in fig. 6–7 shows typical case. Left plot presents daily estimates of baseline length in mm obtained from semi-phase delay solutions and the right plot shows the baseline evolution obtained from group delay solutions. Average baseline length (5040 km) is removed in both plots. Argument in plots is time in days elapsed from 01-JAN-94. Error bars are 1σ formal uncertainties.

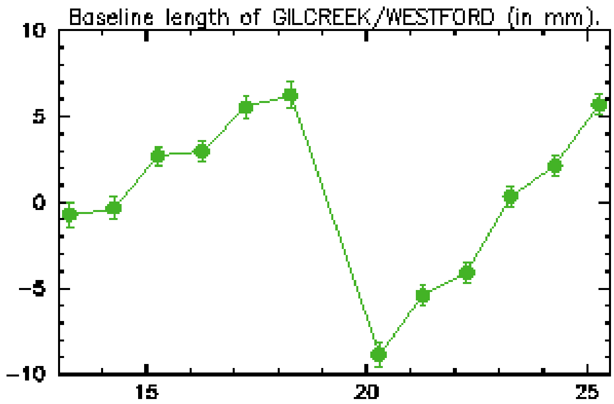

fig. 6

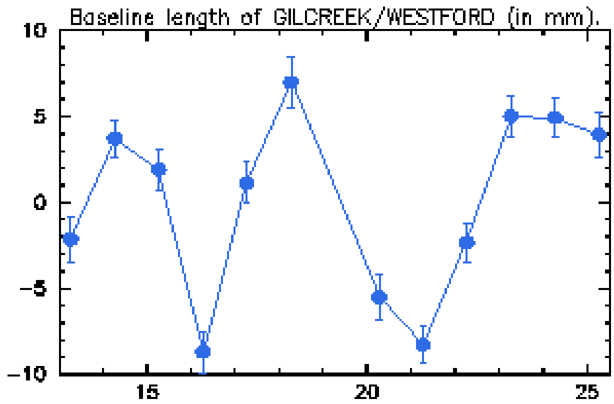

fig. 7

Hourly values of UT1 and pole motion were also obtained. UT1 was modeled by a sum of global offset, global rate and linear spline with 1 hour segment. Rate of change between adjacent segments was constrained with weights to be reciprocal to the constant 0.3 msec/day. The estimates from the phase delay solutions are less noisy and deviate less from the empirical model then the estimates obtained in group delay solutions although an improvement is not conclusive. Results of determination of UT1 for hour intervals using semi-phase and group delay solutions are presented in fig. 8–9 respectively. Argument is time in hours. Mean value and linear trend were removed. A thin solid line represents empirical model derived using a whole set of VLBI group delay observables for 20 years [4]. A grey bar represents 1σ formal uncertainties.

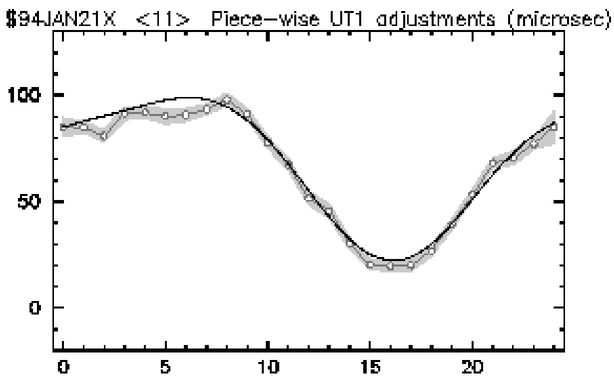

fig. 8

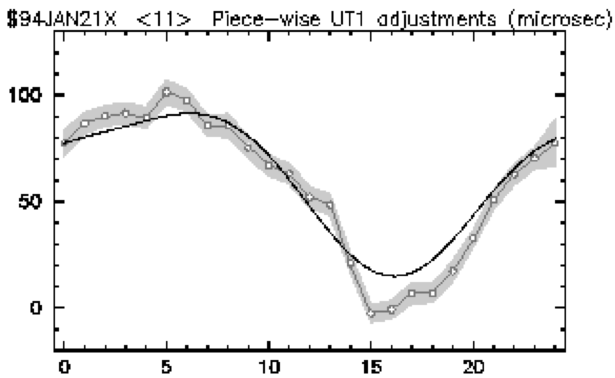

fig. 9

Result of determination of hourly estimates of pole coordinates (in microarcseconds) from semi-phase and group delay solutions are shown in fig. 10–11 respectively.

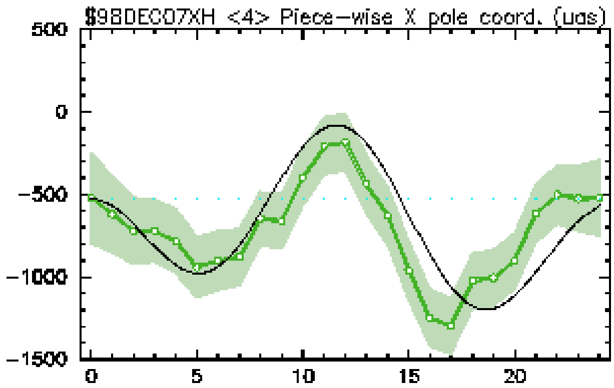

fig. 10

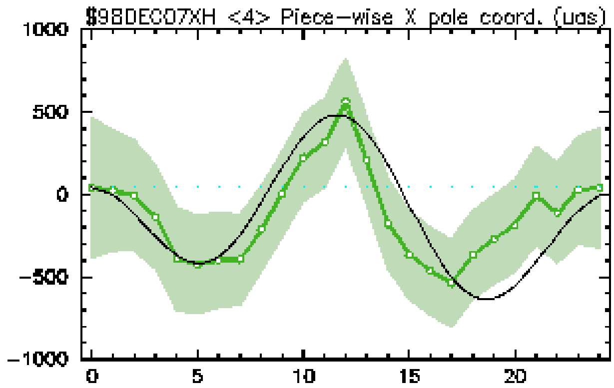

fig. 11

5 Conclusion

- Phase delay ambiguities can be successfully resolved for 80–90% of VLBA observations and for 40–70% of Mark-III data at the baselines with length up to 85% Earth diameter;
- Critically important factors for resolving phase delay ambiguities:
  - High sensitivity: X-band: $\sigma(\tau_{gr}) < 20$psec, S-band: $\sigma(\tau_{gr}) < 100$psec (properly cooled receiver);
  - Low level of instrumental errors (excellent performance of phase calibration).
- Semi-phase delay solutions have not worse precision than pure phase delay solutions, but are more reliable;
- The precision of semi-phase delay solutions is better than the precision of group delay solutions, but an improvement in accuracy is not significant.

6 Acknowledgments

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